# The motions of nearby stars and the local mass density

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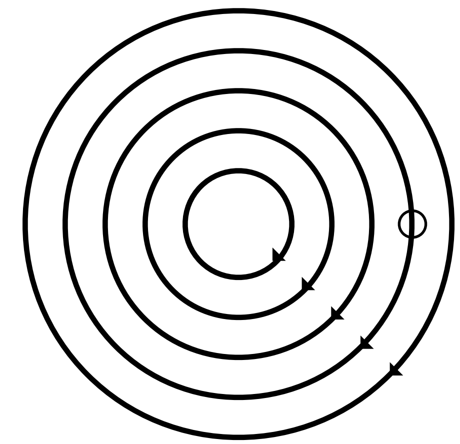
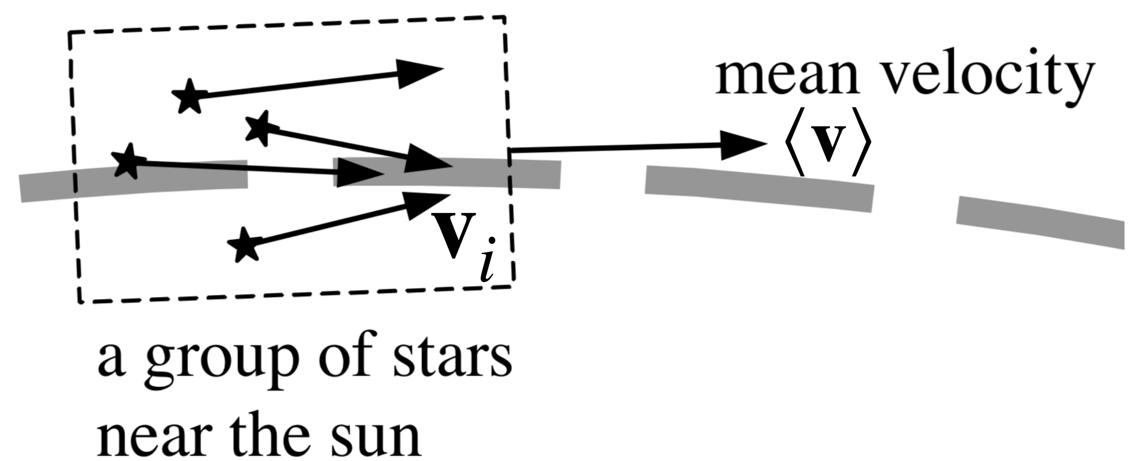
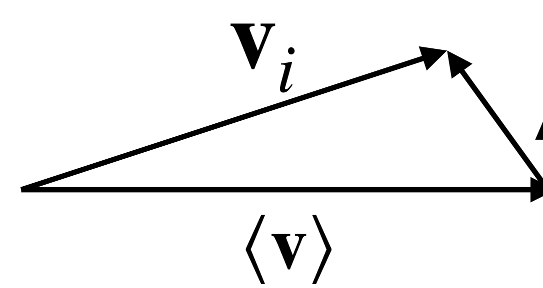
# Motions of nearby stars

- The dominant bulk motion of stars in the Galaxy is their rotation around the Galactic centre on circular orbits
- Stars also have a "random" component to their motion on top of this (the *peculiar motion* $\mathbf{\Delta v}_i$)
- These random motions are especially interesting for an aggregate population: mean velocities and velocity dispersions give a lot of information about dynamics



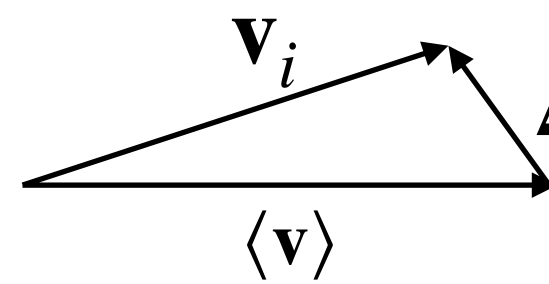



$$\mathbf{\Delta v}_i = \mathbf{v}_i - \langle \mathbf{v} \rangle$$

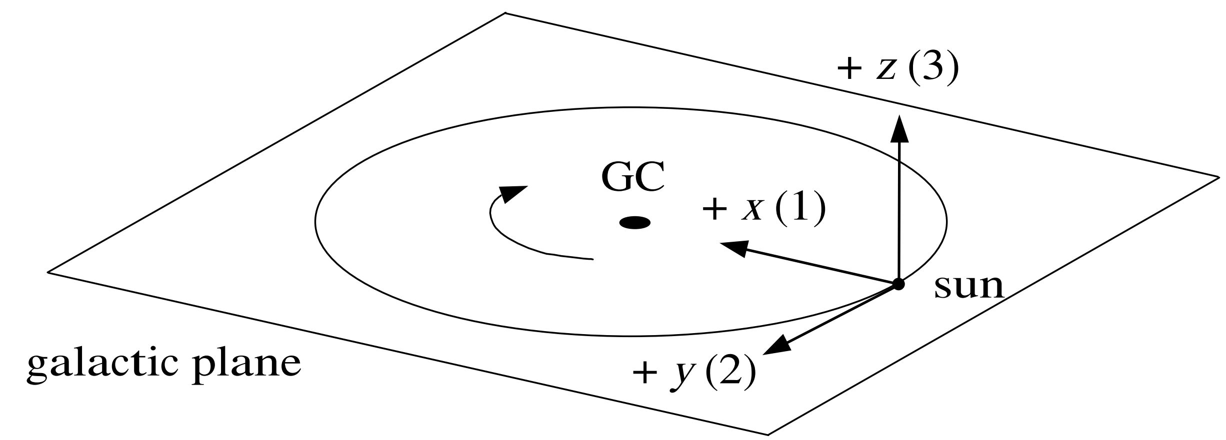
# Quantifying stellar motions

- We work in a heliocentric reference frame, centred on and moving with the Sun




$$\mathbf{\Delta v}_i = \mathbf{v}_i - \langle \mathbf{v} \rangle$$

- Position is denoted $(x, y, z)$ with $x$ pointing towards the Galactic centre, $y$ in the direction of rotation, $z$ out of the disc. Velocity in these directions is denoted $(u, v, w)$




- For a population of $N$ stars, we then have a mean $\langle \mathbf{v} \rangle = \dfrac{1}{N} \sum_{i=1}^{N} \mathbf{v}_i$

- …and a 3x3 symmetric dispersion matrix $\mathbf{D} = \dfrac{1}{N} \sum_{i=1}^{N} \left(\mathbf{\Delta v}_i\right) \left(\mathbf{\Delta v}_i\right)^{\mathrm{T}}$

$$\begin{pmatrix} D_{11} = \sigma_x^2 & D_{12} & D_{13} \\ D_{21} & D_{22} = \sigma_y^2 & D_{23} \\ D_{31} & D_{32} & D_{33} = \sigma_z^2 \end{pmatrix}$$

# Analogy between stellar random motions and kinetic theory of gases

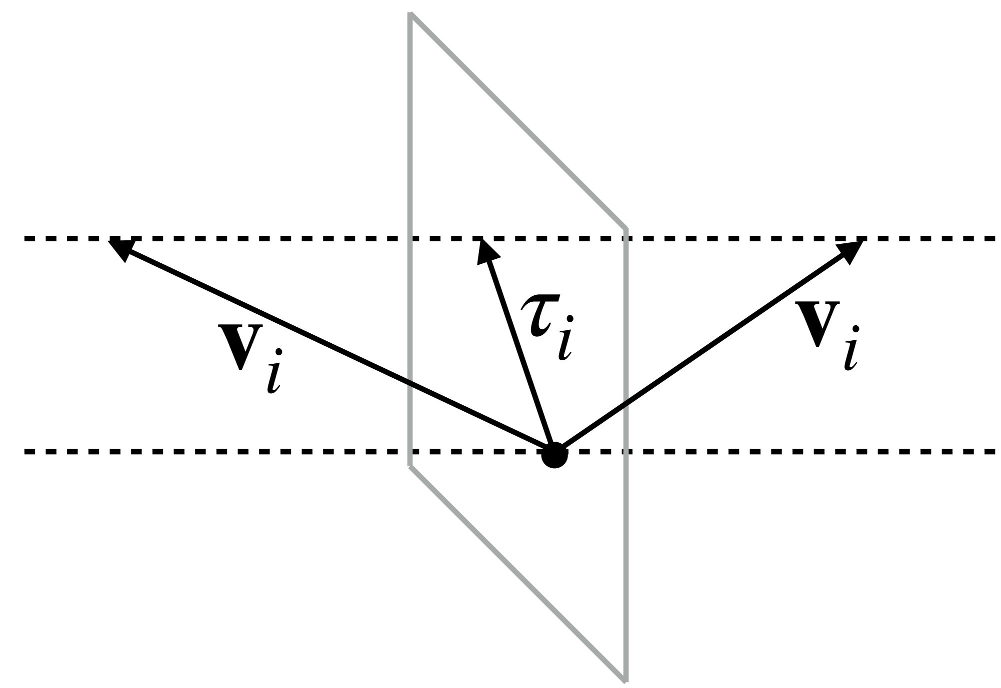
- You can think of stars participating in the Galactic rotation as molecules in a gas: both a bulk motion, as well as the random motions of individual particles
- There are some significant differences: *e.g.*, stars in the Galaxy are *collisionless* ("collision" here meaning a significant gravitational scattering from another star)
- But there are some similarities: *e.g.* we can use the random motions as a measure of internal energy or "temperature"
- For an ideal gas the temperature represents an isotropic velocity distribution on the small scale
- But stars are collisionless, and so can have different velocity dispersions in different dimensions

# What observables do we want? Means and dispersions as a function of colour

- As stellar populations age they get dynamically “heated” by encounters with giant molecular clouds, spiral arms, and other large-scale matter inhomogeneities (not so much star–star encounters)
- Thus, older populations of stars should have higher velocity dispersions
- On the MS, the redder stars are, on average, older
- So you will investigate the stellar velocity distributions as a function of stellar colour
- We sometimes talk of dynamically “hot” and “cold” populations of stars (don’t confuse this “heating” with the stellar effective temperature!)

# How? Can't deproject individual velocities, but can statistically

- We work with proper motions and parallaxes, not radial velocities, since we have more stars with the former than the latter. So for individual stars, we only have 2D motions
- It is of course impossible to "deproject" a single star's velocity vector: $\tau_i = \mathbf{T}_i \mathbf{v}_i$ where $\mathbf{T}_i$ is a singular projection matrix
- But with a large population spread over the sky, we can do so for the averaged quantities, so long as we have reasonable sky coverage and the velocity distribution does not vary with position (Dehnen & Binney 1998)



# Project A

- You will calculate mean velocities (relative to the Sun) and velocity dispersions, and use them to…

Plot $\langle u \rangle$, $\langle v \rangle$, $\langle w \rangle$ as functions of colour index and as functions of the logarithm of the mean age. Conclusions? Do the same for $\sigma_u^2$, $\sigma_v^2$, $\sigma_w^2$. Conclusions?

The ratio $\sigma_v / \sigma_u$ can be theoretically predicted by the epicycle theory. Does the observed value agree with this expectation?

The heating of the stellar disk is often described by a power law of the form $\sigma \propto T^{\alpha}$, where $T$ is the age of the stars and $\alpha$ a 'heating exponent'. This relation is most easily investigated in a log-log plot. Do this and explain the results. Can anything be said about possible heating mechanisms?

Plot $\langle u \rangle$, $\langle v \rangle$, $\langle w \rangle$ as functions of $\sigma_u^2$. What are the relations expected from the asymmetric drift equation? Assuming that the LSR corresponds to the motion of a stellar population with zero velocity dispersion, what is the heliocentric velocity of the LSR? What is the velocity of the sun relative to the LSR? It should be remembered that the kinematics of very young stars may be strongly affected by local effects (which?).

Assuming that the circular velocity is 220 km/s at the sun's distance from the galactic centre, what is the galactocentric velocity of the sun?

Investigate the *vertex deviation*

$$\theta = \frac{1}{2} \arctan \left( \frac{2D_{12}}{D_{11} - D_{22}} \right) \tag{9}$$

as function of colour. The vertex deviation is the galactic longitude of the major axis of velocity ellipsoid, i.e., the direction in which the velocity dispersion is the greatest. For an axisymmetric potential we expect $\theta \simeq 0$. If it consistently deviates from zero for all (or most) colour intervals, it is an indication that the galactic potential is not axisymmetric.

# Epicyclic motions

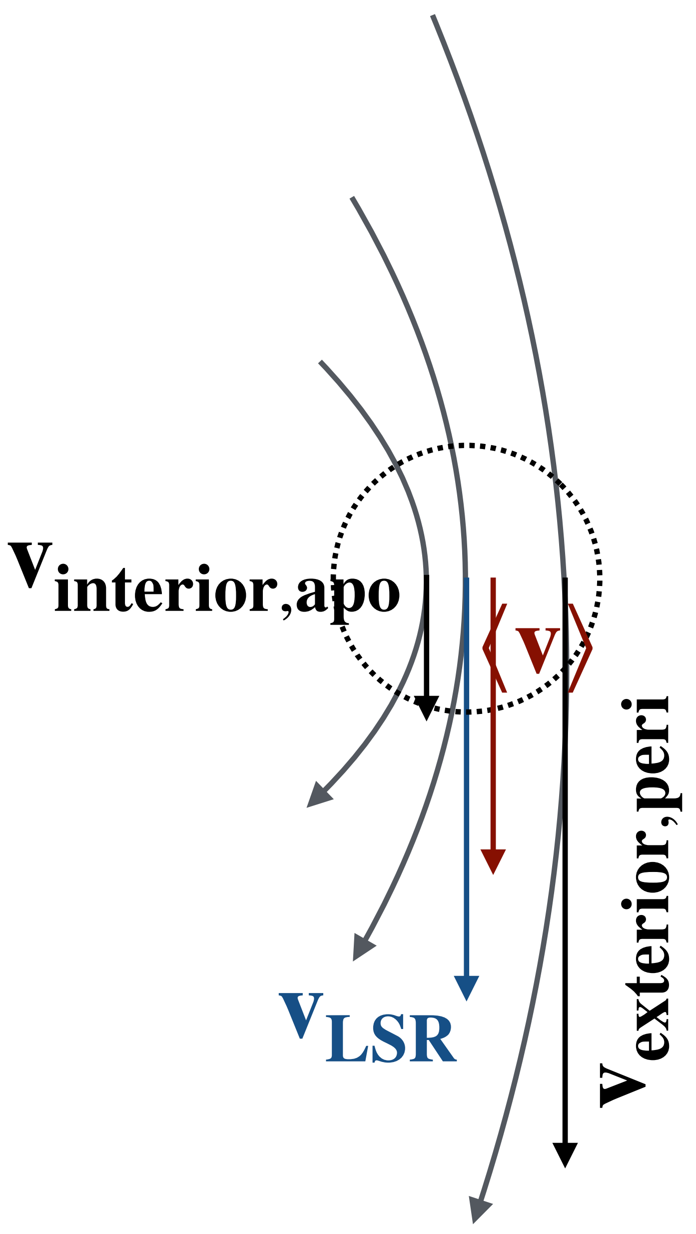
- Most stars orbit the Galactic centre on non-circular orbits
- If the radial motion is small, an epicyclic approximation gives a ratio of $\sqrt{\frac{-B}{A-B}}$ between the amplitudes of oscillation of the azimuthal and the radial velocity components where $A$ and $B$ are the Oort's Constants
- You can therefore compare your epicyclic motions against literature values for the Oort's constants

# Disc heating

- Stellar populations are "heated" as they age
- You can look at the three components of the velocity dispersion as a function of mean age of the stellar populations $\sigma_{u,v,w} \propto T_{\rm age}^{\alpha}$
- Different heating mechanisms give different exponents: can you distinguish between them?
- (see *e.g.*, Binney & Tremaine §8.4, Bland-Hawthorn & Gerhard §5.3)

# Asymmetric drift

- The mean orbital velocity of a stellar population is, in general, not the same as the velocity of a circular orbit (the LSR velocity)!
- The mean velocity is usually slower than the LSR velocity: this is called *asymmetric drift*
- Why is this?
    - there are more stars closer to the Galactic centre
    - stars move more slowly at orbital apocentre
    - so there are more stars at apocentre on interior orbits, than at pericentre on exterior orbits, and they spend more time there
    - Thus, mean velocity lags the circular velocity

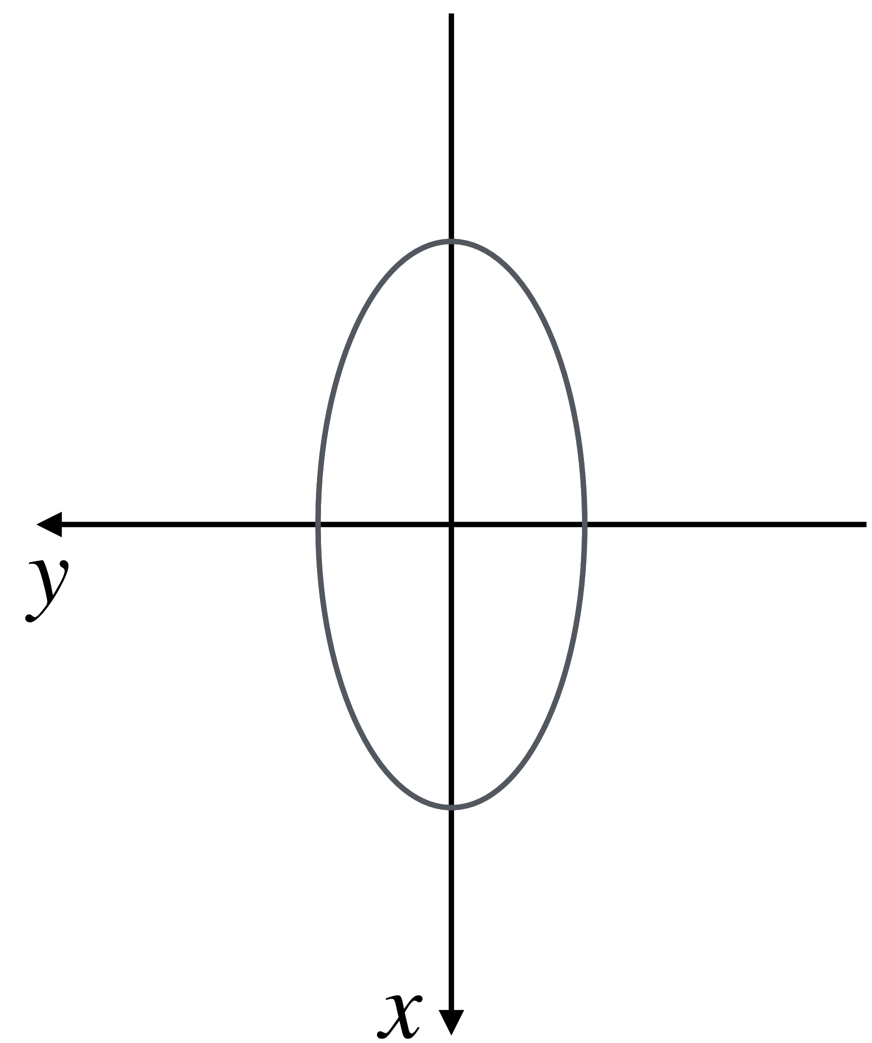


# Velocity of the Sun and the LSR

- From the mean velocities you have the velocity of the Sun relative to the mean velocity
- And from the asymmetric drift you have the velocity of the mean velocity relative to the LSR
- Hence, you have the velocity of the Sun relative to the LSR
- And if you know the LSR velocity (given as 220km/s), you have the velocity of the Sun relative to the Galactic Centre

# Vertex deviation

- If the Galaxy is axisymmetric and in steady state, the velocity ellipsoid should align with the *xy* axes of the reference frame
- The orientation of the ellipsoid is given by the *vertex deviation*
$$\theta = \frac{1}{2}\arctan\left(\frac{2D_{12}}{D_{11} - D_{22}}\right)$$
- If there is a nonzero vertex deviation, one or both of the assumptions is incorrect



# Project B

- You will calculate mean velocities (relative to the Sun) and velocity dispersions, and use them to…

Different samples of stars, selected by their colour index $G_{BP}-G_{RP}$, have different mean ages and therefore systematically different mean velocities and dispersion matrices. In particular the variation of $\sigma_w^2 = D_{33}$ with $G_{BP}-G_{RP}$ (or age) can be analysed in terms of the stellar disk heating.

In the plane parallel approximation (Sect. 10.1), the vertical component of Jeans' equation gives the following estimate of the mass density $\rho_0 = \rho(z \simeq 0)$ in the solar neighbourhood:

$$\rho_0 = -\frac{\sigma_w^2}{4\pi G}\left[\frac{\partial^2 \ln n(z)}{\partial z^2}\right]_{z=0} . \tag{4}$$

$n(z)$ is the number density of some tracer population as function of height ($z$) above the galactic plane, and $\sigma_w$ is the velocity dispersion along the $z$ axis of the same tracer population.

Different tracer populations (as distinguished, e.g., by their colours) have different velocity dispersions and different number density profiles, but they should give consistent estimates of the mass density.

# The local mass density

- You will use stars' vertical motions to determine the local density of matter in the Solar neighbourhood
- We will see on Tuesday that the stellar vertical velocities, number density, and the vertical potential gradient, are related by

$$\langle w \rangle \frac{\partial \langle w \rangle}{\partial z} + \frac{1}{n} \frac{\partial \left( n D_{ww} \right)}{\partial z} = - \frac{\partial \psi}{\partial z}$$

- This is similar to a hydrostatic atmosphere

$$\frac{\partial P}{\partial z} = - \rho g$$

- Stellar number density should increase towards the Galactic midplane

# The local mass density

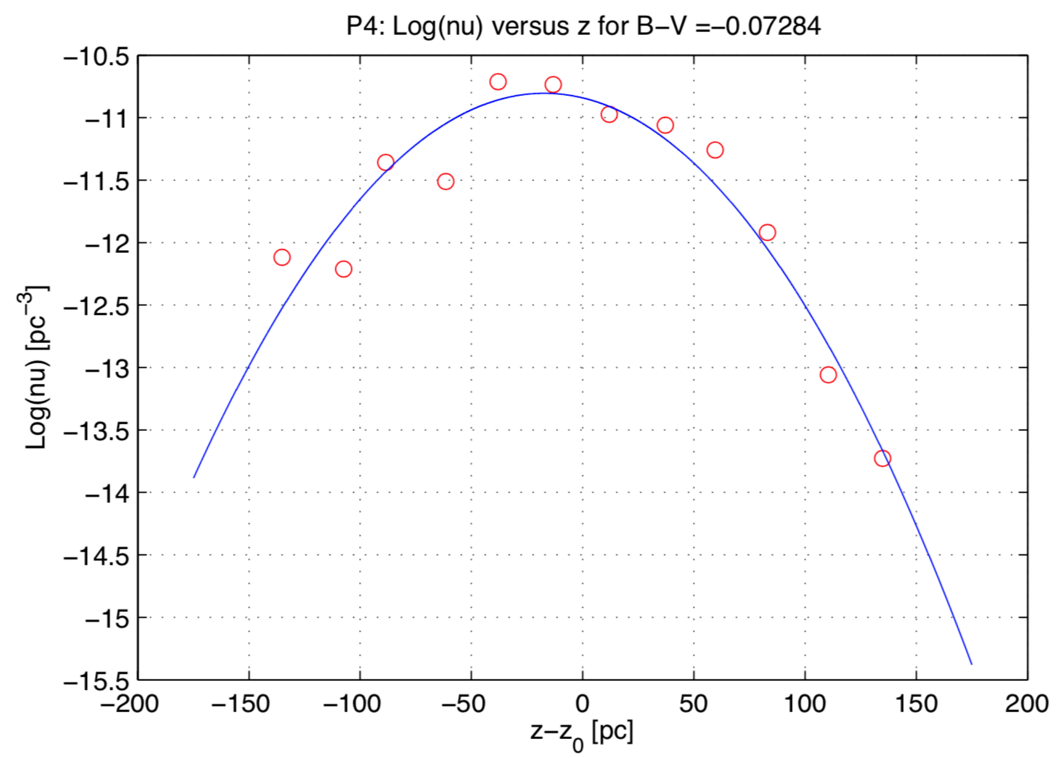
- With some simplifications, and using Poisson's equation, we can show

$$\rho_0 = -\frac{\sigma_w^2}{4\pi G}\left[\frac{\partial^2 \ln n(z)}{\partial z^2}\right]_{z=0} \quad \text{in the midplane}$$

- Here, $n$ and $\sigma_w$ are measured for each stellar population (colour), but $\rho_0$ is the total matter density, the same for all populations: different populations should give the same $\rho_0$
- In this project, you count the number of stars at different $z$-heights above and below the Solar position, to determine the number density as a function of $z$
- Make sure you have a sample that is complete (Eq 3.3)!

# The local mass density

- You then plot and fit a parabola for the different stellar populations
- $$\rho_0 = -\frac{\sigma_w^2}{4\pi G}\left[\frac{\partial^2 \ln n(z)}{\partial z^2}\right]_{z=0}$$
- The curvature, combined with the velocity dispersion, tells you the mass density
- The offset from zero tells you the Sun's location above/below the plane
- These results should be consistent for all your samples



# How do we do this?

- For each star we have $\tau_i = \mathbf{T}_i \mathbf{v}_i$
- $\mathbf{T}_i$ is singular, but for a group of well-spread stars $\left\langle \mathbf{T}_i \right\rangle$ is not
- If positions and velocities are statistically independent, we have $\langle \tau \rangle = \langle \mathbf{T}\mathbf{v} \rangle = \langle \mathbf{T} \rangle \langle \mathbf{v} \rangle$
- This is invertible and $\langle \mathbf{v} \rangle = \langle \mathbf{T} \rangle^{-1} \langle \tau \rangle$
- So we can easily recover the mean 3D velocity of a stellar population
- Velocity dispersions are more tricky…

# How do we do this?

- For each star we have the peculiar motion in the plane of the sky $\boldsymbol{\Delta\tau}_i = \mathbf{T}_i \left( \mathbf{v}_i - \langle \mathbf{v} \rangle \right) = \tau_i - \mathbf{T}_i \langle \mathbf{v} \rangle$
- We then define the outer product $\mathbf{B} = \left\langle \boldsymbol{\Delta\tau\Delta\tau}^{\mathrm{T}} \right\rangle$
- Writing out suffices and using the Einstein summation convention:
  - $\Delta\tau_k = T_{km}\Delta v_m$
  - $B_{kl} = \left\langle T_{km}T_{ln} \right\rangle D_{mn}$
- (Suffix $i$ labels stars; all others labels Cartesian axes)
- This is a tensor equation relating the two second-order tensors $B_{kl}$ and $D_{mn}$ and the fourth-order tensor $\left\langle T_{km}T_{ln} \right\rangle$

# How do we do this?

- $B_{kl} = \left\langle T_{km} T_{ln} \right\rangle D_{mn}$
- We can solve this using Numpy linear algebra routines
- (Or turn it into a normal matrix equation: less elegant)
- Use numpy.einsum to compute $\left\langle T_{km} T_{ln} \right\rangle$
- And then numpy.linalg.tensorsolve to solve the equation

# Getting started on the project

- You have a Jupyter notebook with a *Gaia* ADQL query and the linear algebra to compute the velocity dispersion already implemented
- You will want to think about how you select your samples: cuts in parallax, parallax error, colour bins, *etc*.
- The query provided gives quite a small sample: you can test your code on this then go fainter when you're sure everything is working
- Note that if you go too faint your *Gaia* query will cut off at 3 million stars without warning, so check this isn't happening!
- You should start off by making an HR diagram of your sample to see what stars you're working with and what kind of cuts will you make (do you want giants, MS stars, *etc*.?)
- You'll need to estimate stellar ages by colour bin: it suffices for this project to take half the turn-off age which you can estimate from Fig 5 and 6 in the booklet