Use of BAM data in AGIS

Abstract

This note describes how BAM (Basic Angle Monitor) data will be incorporated in the core astrometric solution (AGIS).
Document History

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The need for BAM data in AGIS

The use of a very stable basic angle ($\Gamma$) between the two viewing directions of Gaia is fundamental for its ability to construct a globally consistent reference frame (i.e., without large-scale distortion in the system of positions and proper motions), as well as for the determination of absolute parallaxes (Lindegren & Bastian, 2011). The requirement on the basic angle stability is specified in the ESA Gaia Project (GAIA-EST-RD-00553) as:

\[ \text{SCI-280} \quad \text{The basic angle fluctuations over the nominal spin period shall be:} \]
\[ < 7 \, \mu\text{as rms for the random contribution} \]
\[ < 4 \, \mu\text{as for the systematic contribution.} \]

The word systematic has to be understood as the amplitude of the fluctuations at the spacecraft spin frequency (inverse spin period) and at the low-order multiples of this frequency.

Note that SCI-280 does not specify any requirement for the stability of the basic angle over longer time scales than one spin period (6 hr). This is because the astrometric solution for primary sources (using AGIS) in principle allows to determine the average of $\Gamma(t)$ over any time interval longer than the spin period $P = 6 \, \text{hr}$. In practice such averages will be computed with a time resolution of the ‘short’ geometric calibration time interval (see Sect. 2) which will be at least a few spin periods.

Requirement SCI-280 is basically achieved by the passive temperature stabilization of the optomechanical structure, and is sufficient to achieve the primary science goal of a parallax accuracy around 10 $\mu\text{as}$ for bright stars. However, as explained in Lindegren (GAIA-LL-057), there are compelling scientific reasons to keep the systematic errors at a very much lower level than the random errors, perhaps by as much as two orders of magnitude (i.e., on the 0.1 $\mu\text{as}$ level). This applies in particular to the global parallax zero point, which is perfectly correlated with (and therefore indistinguishable from) a particular signal in $\Gamma(t)$. Specifically, observations obtained with $\Gamma(t) = \Gamma_0 + a_1 \cos \Omega$, where $\Gamma_0$ and $a_1$ are constants and $\Omega$ is the azimuth angle of the sun in the instrument (SRS) system, are indistinguishable from observations obtained with $\Gamma(t) = \Gamma_0$ if, at the same time, the parallaxes of all the sources are shifted by the amount $0.874 a_1$ (Butkevich et al., 2010).

Other errors signals in $\Gamma(t)$, at frequencies above $1/P$, may also produce systematic astrometric errors on a level comparable to the amplitude of the variations (see the discussion in Sect. 8). Due to the thermal inertia of the optomechanical structure, it is however expected that the amplitude spectrum of these signals is a steeply decreasing function of frequency.

The conclusion is therefore that we need to control variations at least up to several times the spin frequency $1/P$ to a level of about 0.1 $\mu\text{as}$.

\[ ^1 \text{The meaning of the ‘average’ in this context has not been precisely defined. Presumably it refers to some kind of weighted average after having removed the component that is degenerate with respect to the parallax zero point (explained in the next paragraph).} \]
This cannot be achieved by the passive stabilization. The adopted solution is to use an interferometric device, the Basic Angle Monitor (BAM), to measure short-term variations of the basic angle, and subsequently correct the observations. The requirement on the BAM performance is (ESA Gaia Project, GAIA-EST-RD-00553):

SCI-290 The Basic Angle shall be monitored in flight with accuracy better than 0.5 μas rms for every 5 minutes interval of scientific operation.

If the thermal time constant is at least 2 hr, it appears that SCI-290 should be sufficient to guarantee sufficient knowledge of the short-term variations of the basic angle.

It is possible to introduce the measured basic-angle variations simply as corrections to the observations, which can then be treated as if they had been obtained with a stable basic angle. However, as will be explained below, the preferred method, at least initially, is to introduce the correction indirectly via the calibration solution in AGIS. This provides a safeguard against possible errors in the BAM calibration as well as important diagnostic information, e.g., on the relevance of the BAM signal for the astrometric data. Nevertheless, for the final astrometric solution it may be necessary to adopt a slightly different approach, as discussed in Sect. 8.

2 What is the basic angle?

The precise definition of the basic angle is quite involved and intimately connected to the geometric instrument calibration model as well as the corresponding data processing in AGIS. Conceptually, the basic angle \( \Gamma \) is the angle between the viewing directions in the preceding and following fields of view. However, for practical reasons the viewing directions are taken to be fixed in the Scanning Reference System (SRS; Bastian, BAS-003), which implies a fixed basic angle. Indeed, in the current formulation of the geometric calibration model (Lindegren et al., 2011), the basic angle is a purely conventional value and the actual deviation, as well as its temporal variation, is described by the large-scale geometric calibration parameters. However, this only covers the variations on time scales longer than the spin period, or several times the spin period.

Thus, we need to distinguish between the above-mentioned conventional basic angle \( \Gamma_c \) (which by definition is constant), the actual basic angle \( \Gamma(t) \), and the basic angle represented by the large-scale geometric calibration parameters, here denoted \( \Gamma_j \). The latter is constant in each ‘short’ calibration time interval \( j \) (say from \( t_j \) to \( t_{j+1} \)) and is given by

\[
\Gamma_j = \Gamma_c + \Delta \Gamma_j = \Gamma_c + \frac{1}{62} \sum_{n \in AF} \sum_{f} f \Delta \eta_{0fn0j}
\]

(Lindegren et al., 2011), where \( \Delta \eta_{0fn0j} \) is the zeroth-order along-scan (AL) large-scale calibration parameter for field index \( f \) (±1 in preceding/following field), CCD index \( n \) (for the 62
CCDs in the astrometric field, AF), gate 0 (i.e., no gating), and calibration time interval \( j \). We assume that \( \Gamma_j \) equals, to a very good approximation, the average actual basic angle

\[
\Gamma_j = \langle \Gamma(t) \rangle_j ,
\]

where \( \langle \rangle_j \) denotes the unweighted average over the calibration time interval \( j \). The basic-angle variations within the calibration time interval may be represented by the function

\[
\delta \Gamma(t) = \Gamma(t) - \Gamma_j , \quad t_j \leq t < t_{j+1} ,
\]

which in general is discontinuous at the interval division times \( \ldots , t_{j-1} , t_j , t_{j+1} , \ldots \).

It should be noted that other geometric calibration parameters, such as the higher-order large-scale parameters \( \Delta \eta_{r_{fr0j}} \) (for \( r > 0 \)) and the small-scale AL calibration parameters \( \delta \eta_{n0km} \) (where \( k \) is the ‘long’ calibration time interval and \( m \) the pixel column index), describe deviations that are orthogonal to the large-scale calibration and therefore need not be considered here. It can also be noted that \( \Gamma_j \) is entirely defined in terms of the AL calibration for ungated observations of primary sources.

### 3 What are the BAM data?

The Basic Angle Monitor (BAM) is assumed to measure the variations of the true basic angle \( \Gamma(t) \) on short time scales (minutes to days). Although the measurements are of course provided at discrete times (with a maximum sampling interval of 5 min according to SCI-290), they are here represented by the continuous function \( h(t) \). We shall assume the following model for the BAM signal:

\[
h(t) = a(t) + b(t) [\Gamma(t) - \Gamma_c] + \nu_{\text{BAM}}(t) ,
\]

where \( a(t) \) and \( b(t) \) are offset and scale functions that change only very slowly with time (see below), and \( \nu_{\text{BAM}}(t) \) is measurement noise with zero mean value and unknown (but very small) standard deviation.

The notion that BAM measures the variations of \( \Gamma(t) \) on short time scales is captured by the following assumptions about the functions \( a(t) \) and \( b(t) \):

- That BAM only measures variations on a short time scale implies that it could have a long-term drift, which is described by the slowly varying offset function \( a(t) \). For example, \( a(t) \) might be adequately represented by a linear or quadratic function of time over some relatively long time interval (many days). Alternatively, \( a(t) \) could be regarded as constant over any moderately long time interval. Possibly, it could have physical discontinuities at specific (and known) times, e.g., when some major anomaly occurs in the satellite operations. For simplicity, we will assume that \( a(t) \) is adequately represented by a constant value, \( a_j \), in each ‘short’ calibration time interval.
• The scale factor \( b(t) \) should, in principle, be known from the design (and on-ground calibration) of the BAM. Moreover, since it depends only on the wavelength of the BAM laser and the geometrical configuration of the BAM, both of which are expected to be very stable throughout the mission, it could be represented by a constant \( b \). Laser mode hopping (which should not happen) or a switch from the nominal to the redundant BAM would however cause a sudden change in \( b \). For the present analysis we assume a constant \( b \) for the whole mission, although it would be a trivial modification to assume a few different discrete values, provided that each discontinuity coincides with some division \( t_j \) in the sequence of short calibration time intervals.

According to these assumptions we can simplify Eq. (4) to read

\[
h(t) = a_j + b [\Gamma(t) - \Gamma_c] + \nu_{BAM}(t) \tag{5}
\]

if \( t \) belongs to interval \( j \). The average BAM signal in interval \( j \) is

\[
h_j \equiv \langle h(t) \rangle_j = a_j + b [\Gamma_j - \Gamma_c] + \nu_{BAM}^j,
\tag{6}
\]

where \( \nu_{BAM}^j \equiv \langle \nu_{BAM}(t) \rangle_j \) is the BAM noise averaged over the calibration time interval. Introducing, in analogy with Eq. (3), the relative BAM signal

\[
\delta h(t) = h(t) - h_j
\tag{7}
\]

and the relative measurement noise

\[
\delta \nu_{BAM}(t) = \nu_{BAM}(t) - \nu_{BAM}^j \tag{8}
\]

we have from Eqs. (5) and (6):

\[
\delta h(t) = b \delta \Gamma(t) + \delta \nu_{BAM}(t) \tag{9}
\]

4 Calibration model including BAM data

The generic calibration model for the astrometric processing (Lammers [UL-031]) can be written

\[
\eta_{fng}(\mu \mid c) = \eta_{ng}^0 + \sum_{r=0}^{N_{AL} - 1} \sum_{s=0}^{K_r - 1} c_{rs} \Phi_{rs} \tag{10}
\]

(Lindegren et al., 2011), where \( \eta_{fng}(\mu \mid c) \) is the observation line for CCD number \( n \) and gate \( g \) as function of the AC pixel coordinate \( \mu \), and \( \eta_{ng}^0 \) is the corresponding nominal field angle of the CCD/gate combination. \( c_{rs} \) (summarized in the array \( c \)) are calibration parameters and \( \Phi_{rs} \) are elemental calibration functions\footnote{See Lindegren et al. (2011) for further explanation of the compound index \( rs \).} uniquely computable as functions of various data associated

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with an observation (such as its time, AC coordinate, etc.). With \( l \) denoting the observation counter, we may thus regard \( \Phi_{rs} \) as a function of \( l \). We now add an explicit term to take into account the basic angle variations as measured by the BAM data:

\[
\eta_{fng}(\mu | c) = \eta_{fng}^0 + \sum_{r=0}^{N_{sl}-1} \sum_{s=0}^{K_r-1} c_{rs} \Phi_{rs}(l) + \frac{1}{2} c_{BAM} f \delta h(t_l),
\]

where \( f = \pm 1 \) is the field-of-view index as before, \( c_{BAM} \) is the new BAM calibration parameter, and \( \delta h(t_l) \) is the relative BAM signal from Eq. (9) interpolated to the time of the observation. The sign on the last term is positive because an increase in the physical basic angle (positive \( \delta h(t_l) \)) results in an increased observed \( \eta \) value in the preceding field \( (f = +1) \) and a decreased value in the following field \( (f = -1) \), as measured relative to the field centre defined by the (fixed) conventional basic angle.

Formally \( c_{BAM} \) is a calibration parameter just like \( c_{rs} \), and should therefore be considered part of the vector \( c \); however, we return in Sect. 7 to the question how it should be treated in the astrometric solution.

5 Expected solution for the BAM calibration parameter

It is instructive to derive an approximate expression for the expected value of \( c_{BAM} \) obtained in the astrometric solution (AGIS), as a response to a given BAM signal. To this end it is necessary to make some breakneck assumptions, for example that \( c_{BAM} \) is the only parameter to be solved in AGIS.

The astrometric solution minimizes the weighted sum \( Q = \sum_l W_l R_l^2 \) of the squares of the observation residuals

\[
R_l = \eta_{fng}(\mu_l | c) - \eta(t_l | s, a, g)
\]

Lindegren et al. (2011), where the first term is the ‘observed’ \( \eta \) coordinate defined as in Eq. (11) in terms of the calibration parameters \( c \) and the AC coordinate \( \mu_l \) of the observation, and \( \eta(t_l | s, a, g) \) is the AL field angle computed for the source at the time of observation \( (t_l) \) for the given source \( (s) \), attitude \( (a) \) and global \( (g) \) parameters. \( W_l \) is the statistical weight of observation \( l \).

If we assume (i) that there are no basic angle variations whatsoever, (ii) that \( c_{BAM} \) is set to zero, and (iii) that all other parameters are perfectly adjusted, then the residuals in Eq. (12) essentially represent the observation noise, which we denote \( \nu_{\text{obs}} \). Now, if we introduce real basic angle variations according to Eq. (3), but keep all parameters (including \( c_{BAM} \)) at the same values as

\[\text{lindegren et al. 2011}\]

3The term ‘interpolated’ should here be interpreted in a very general sense: it could mean anything from looking up the nearest BAM reading to computing a frequency-filtered value (see Sect. 8 for a discussion about filtering).
before, the residuals will be
\[ R_l = \nu_l^{\text{obs}} - \frac{1}{2} f \delta \Gamma(t_l) . \] (13)

The sign on the second term is negative because a positive \( \delta \Gamma \) leads, in the preceding field \( \delta f = +1 \), to an earlier observation (smaller \( t_l \)) and consequently (since \( d\eta/dt < 0 \)) a larger computed field angle \( \eta(t_l | s, a, g) \); the ‘observed’ field angle \( \eta_{\text{ang}}(\mu_l | c) \) is not affected by the change in \( \Gamma \).

The solution will try to compensate this change in \( R_l \) by means of the BAM calibration term in Eq. (11). Assuming that only \( c^{\text{BAM}} \) needs adjustment, the solution thus minimizes the weighted sum of the squares of
\[ R_l = \nu_l^{\text{obs}} - \frac{1}{2} f \delta \Gamma(t_l) + \frac{1}{2} f c^{\text{BAM}} \delta h(t_l) . \] (14)

Putting
\[ \frac{\partial Q}{\partial c^{\text{BAM}}} = 2 \sum_l W_l R_l \frac{\partial R_l}{\partial c^{\text{BAM}}} \] (15)
equal to 0 and solving for \( c^{\text{BAM}} \) gives the estimate
\[ c^{\text{BAM}} = \frac{\sum_l W_l \delta \Gamma(t_l) \delta h(t_l) - 2 \sum_l W_l f \nu_l^{\text{obs}}}{\sum_l W_l [\delta h(t_l)]^2} . \] (16)

The second sum in the nominator will be zero because otherwise the residuals \( \nu_l^{\text{obs}} \) would require a change in \( \Delta \Gamma_j \), which is contrary to our assumptions above. In the other two sums we insert the BAM signal model from Eq. (9) and assume that the BAM noise is statistically uncorrelated with the basic angle fluctuations,
\[ \sum_l W_l \delta \Gamma(t_l) \delta \nu^{\text{BAM}}(t_l) = 0 . \] (17)

The result is
\[ c^{\text{BAM}} = \frac{b \sum_l W_l [\delta \Gamma(t_l)]^2}{b^2 \sum_l W_l [\delta \Gamma(t_l)]^2 + \sum_l W_l [\delta \nu^{\text{BAM}}(t_l)]^2} . \] (18)

Although all the sums are weighted by \( W_l \), and therefore depend somewhat on the actual distribution of the observations, they are essentially like time averages (over the whole mission, or extended parts of the mission, see Sect. 7) of the squared functions \( \delta \Gamma(t) \) and \( \nu^{\text{BAM}}(t) \). Using angular brackets \( \langle \rangle \) for these mission averages, we have
\[ c^{\text{BAM}} = b^{-1} \frac{\langle \delta \Gamma^2 \rangle}{\langle \delta \Gamma^2 \rangle + \langle (\delta \nu^{\text{BAM}}/b)^2 \rangle} , \] (19)

where \( \delta \nu^{\text{BAM}}/b \) is the BAM noise expressed in angular units. Equation (19) has a simple interpretation. In the absence of BAM noise we find \( c^{\text{BAM}} = b^{-1} \) as would naively be expected from
the BAM signal model and the definition of $c^{\text{BAM}}$. However, in the presence of BAM noise, $c^{\text{BAM}}$ is reduced by the factor\[ F = \frac{S^2}{S^2 + N^2}, \]where $S = (\delta \Gamma)_{\text{RMS}}$ is the RMS ‘signal’ and $N = (\delta \nu^{\text{BAM}}/b)_{\text{RMS}}$ the RMS ‘noise’ in the BAM data, both expressed in angular units. We always have $0 \leq F \leq 1$, with $F = 0$ in the limit of very large noise (or no signal), and $F = 1$ in the limit of noiseless data. $F$ is optimized, depending on the signal-to-noise ratio ($S/N$) of the BAM data, to minimize the sum of the squared residuals in the solution. In Sect. 8 we will discuss if this is what we want.

Since in any case $c^{\text{BAM}}$ is inversely proportional to $b$ it follows that the result is independent of a scaling error in the BAM data ($\delta h(t)$). Indeed, even if the data have the wrong sign the only effect is that $c^{\text{BAM}}$ will also get the opposite sign, but the correction applied to the data is exactly the same. Thus the method automatically provides a safeguard against possible scaling or sign errors in the BAM calibration – in fact, we need not even know exactly what $h(t)$ represents in order to use it in the AGIS solution.

Moreover, in the absence of a real BAM signal ($S = 0$), the result will be $c^{\text{BAM}} = 0$. This means that the method described here will automatically remove the BAM noise from the modelling of the observations if there is nothing to be gained from the BAM measurements.

6 Which basic angle errors can be recovered in AGIS?

The solution for $c^{\text{BAM}}$ will be singular if in Eq. (15) $\partial R_l/\partial c^{\text{BAM}} = 0$ for all $l$, i.e., if the residual in Eq. (12) is independent of the BAM signal. As explained in Sect. 1 this will happen if the true basic angle variations are of the form $a_1 \cos \Omega$, in which case the parallaxes will be biased without any visible effect in the residuals. That is, $\delta \Gamma(t) = a_1 \cos \Omega$ is degenerate with the astrometric parameters.

The calibration of $c^{\text{BAM}}$ in AGIS obviously requires that $\delta \Gamma(t)$ has at least some non-negligible component that is not degenerate with the astrometric solution, i.e., that would create an effect in the residuals if left uncorrected. The $S$ value in Eq. (20) should be interpreted as the RMS value of these (non-degenerate) components of the basic angle variation.

The analysis carried out by Klioner & Butkevich in the context of the velocity determination using stellar aberration (presentation at AGIS#13, May 2010) shows that any error pattern in the source parameters can be described by a change in $\Gamma(t)$ (equivalent to the aberration effect of a velocity error along the $x$ axis) and a rotation around the $y$ axis (absorbed by the attitude determination), and is therefore degenerate with the astrometric solution. Also any time-dependent error pattern would be equivalent to some basic angle variation. The $\cos \Omega$ variation is therefore not unique: there is in principle an infinite set of other components in $\delta \Gamma(t)$ that are also
indeterminable from the observations alone. Clearly the BAM is needed also for correcting these.

From the analysis above it would appear to be an almost hopeless task to calibrate the BAM data in AGIS, but that is not the case. Although any error pattern in the source parameters mimics some basic angle error, the reverse is not true: for a given variation $\Gamma(t)$ it is in general not possible to find an astrometric error pattern that leaves the residuals unchanged. In particular, the components of $\Gamma(t)$ that are degenerate with the astrometric solution are in general quasiperiodic with a fundamental frequency close to the sidereal spin rate (the notable exception is the $\cos \Omega$ variation, which is periodic with respect to the direction to the sun). In particular, variations with a frequency significantly below $1/P$ cannot be degenerate.

For example, a nearly linear variation of $\Gamma(t)$ is expected in the initial cool-down phase of the mission, and after each lunar eclipse. It seems obvious that this can be covered in AGIS, and it could be used to calibrate the BAM parameter $c^{\text{BAM}}$.

Following the ideas developed by Klioner & Butkevich (AGIS#13) it appears that most of the frequencies above $1/P$ are also in principle non-degenerate, so that for example random basic angle variations would also be useful for the BAM calibration. However, the precise conditions for this remain to be explored.

One further remark should be made based on the analysis by Klioner & Butkevich. As they demonstrated, any basic angle variation is observationally indistinguishable from the aberrational effects of some specific velocity error along the $x$ axis in the instrument (SRS) system.\textsuperscript{4} Consequently, it is in general not possible to calibrate the BAM data if, at the same time, we want to estimate the velocity correction from the aberrational effect.

### 7 Implementation issues

In principle the BAM term in Eq. (11) should be treated as any other geometric calibration term. In particular it should be included when calculating the Field Angle Offset (FAO; Lindegren LL-089).

However, in the generic calibration scheme the term is still awkward because it is a common parameter for all the CCDs in the Astrometric Field. The geometric calibration is set up to be able to process each CCD (or row of CCDs) independently, except for some normalizing operations.

\textsuperscript{4}The aberrational effects of the $y$ and $z$ components of the velocity error are degenerate with the attitude determination and therefore also unobservable. Strictly speaking, all these degeneracies only hold for an infinitesimal field of view, but the differential effects within the actual field are too small to lift the degeneracy in any practical situation.
There are two main options for handling $c^{\text{BAM}}$ in the generic calibration scheme:

1. Extend the generic calibration scheme to allow (a small number of) calibration parameters that are common to all CCDs. Doing this rigorously might be rather difficult, because it would couple together all the calibration units in the solution. However, if these couplings are simply ignored (they are expected to be small), it is clearly feasible. For example, $c^{\text{BAM}}$ could be added as a ‘normal’ calibration parameter for every CCD. When all the observations have been processed, the 62 matrices are solved, yielding one estimate of $c^{\text{BAM}}$ for every CCD. The adopted value would then simply be a mean value of the 62 estimates.

2. Treat $c^{\text{BAM}}$ as a global parameter in the solution. This means that the normal equations are accumulated, and the parameter solved in the global update block simultaneously with other global parameters. In the formalism of Eq. (12) it should however still be part of $c$ rather than $g$; that is, it will not affect the field angle calculation in $\eta(t \mid s, a, g)$. One argument in favour of treating $c^{\text{BAM}}$ as a global parameter is that its correlations with other global parameters would be calculated as part of the global update process, which is clearly interesting.

For the time being it is left undecided which of the two options will be chosen for the implementation.

It has been assumed that $c^{\text{BAM}}$ is constant throughout the mission, but of course it is also possible to solve it separately for a number of time intervals. The ‘long’ calibration time intervals used for the small-scale geometric calibration are probably adequate for this.

Independent of the above issues, it is necessary that each ‘short’ time interval $j$ covers a continuous series of BAM data, i.e., that no interruptions or discontinuities occur within the interval. If there are such effects in the BAM data, then it is necessary to introduce a time interval division at that point.

8 Discussion

The introduction of the BAM parameter $c^{\text{BAM}}$ as an unknown in the AGIS calibration has some important advantages. First of all, it provides a nice way of checking that the BAM data are really relevant for the astrometric processing – namely, that it allows to reduce the RMS residual of the solution. If that is not the case, the solution will simply return $c^{\text{BAM}} = 0$. Secondly, as already pointed out, it provides a safeguard against BAM calibration errors: even a trivial sign error would easily be detected and automatically corrected.

The $F$ factor in Eq. (20) means that less than the whole correction is applied for a finite $S/N$ ratio. From one point of view this is fine: it is in fact optimal in the sense of minimizing the
total RMS errors of the astrometric data. However, the concern in Sect. 1 was primarily for systematic errors, and if there is for example a periodic variation inducing a parallax zero point error, we want to eliminate that completely, not just part of it. Probably we want to do this even at the expense of a very marginally increased RMS error.

Of course, once the general relevance of the BAM data for the astrometric solution has been verified (e.g., by obtaining a value of $b e^{BAM}$ only slightly below unity), we can fix $c^{BAM} = b^{-1}$ for a final iteration of the source parameters and thus fully eliminate the measured variations.

In any case it is advisable to ensure a high $S/N$ ratio in the BAM data. This can be achieved by pre-filtering the BAM data. To illustrate the point, let us assume that the actual short-term BAM variations have an RMS value of $S = 1 \mu\text{as}$ and that they are measured every 5 min with an RMS error of $0.5 \mu\text{as}$. If in Eq. (11) we use the nearest measured value for $\delta h(t_i)$, then the RMS noise is $N = 0.5 \mu\text{as}$ and we find $F = 0.8$, which means that 20% of the basic angle variation remains uncorrected. Thus, if there is in addition a $\cos \Omega$ component of amplitude $a_1 = 1 \mu\text{as}$ in the basic angle variations, then the residual (uncorrected) parallax bias would amount to $0.2 \times 0.874 a_1 = 0.17 \mu\text{as}$. Now suppose that the BAM data are low-pass filtered so that only the low-order harmonics of the spin frequency ($1/P$) are fully transmitted. If the cut-off frequency is set at $2/P$, for example, the bandwidth is reduced by a factor 18 and $N^2$ is reduced by the same factor. As a result, $F = 0.986$ and the uncorrected parallax bias would only be $0.01 \mu\text{as}$. As pointed out in Sect. 3, higher frequencies might however also be a concern, in which case a less strongly filtered BAM signal should be used for the final correction.

In conclusion, the inclusion of BAM data in the AGIS calibration model allows a safe and simple way to make the best of the BAM data, although some implementation details need to be worked out. Also, the method for applying the final correction in AGIS due to the BAM signal (e.g., the adopted pre-filtering) may depend on the actual characteristics of the signal.

9 Summary of the procedure

The procedure for using BAM data in AGIS can be summarized in the following steps.

1. It is assumed that BAM data are provided in the form of data pairs $(t_i, h_i)$, where $t_i$ is the time of the $i$th measurement and $h_i$ is the corresponding measurement. Although the data need not be equidistant in time, they should form a quasi-regular time series except for known (and specified) data gaps. The measurements should be presented at the highest available time resolution without applying any smoothing or interpolation. Obviously, if a discontinuity could be expected from the operation of the BAM, the corresponding time must also be known.

2. As part of the pre-processing for AGIS, the BAM data are chopped up in segments corresponding to the short calibration time intervals. If there are significant gaps
or other upsets in the BAM data, a corresponding division must be introduced. For each time interval $j$, the relative BAM signal is computed according to Eq. (7). A method is set up to find $\delta h(t)$ for arbitrary $t$, e.g., by looking up the nearest value. If pre-filtering of the BAM data will be used, it should also be part of this pre-processing.

3. The BAM calibration term is added to the calibration model according to Eq. (11) and $c^{BAM}$ is estimated in AGIS. The resulting AGIS solution takes into account the BAM data and the procedure could therefore stop here. However, the subsequent course of action might depend on the actual value obtained for $c^{BAM}$, as discussed in Sect. 8 as well as on any additional diagnostic information, for example if separate BAM parameters are estimated for different segments of the mission.

Having determined $c^{BAM}$ as described above, the resulting parameter(s) together with the BAM data needed to define $\delta h(t)$ will be part of the geometric calibration used to compute the Field Angle Offset (FAO) downstreams, e.g., in the IDU.
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