Modelling radiation damage effects for Gaia:
A first-order Charge Distortion Model (CDM-01)

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Abstract. This note describes a simple analytical Charge Distortion Model (CDM) that may be adapted and further developed for the needs of GIBIS and IDT/IDU. In general, the purpose of the CDM is to describe the distortion of the charge image resulting from radiation damage to the CCD, in terms of the illumination history and a number of adjustable model parameters. The particular model specified here is referred to as CDM-01.

1 Introduction

All astrometric, photometric and spectroscopic CCD data produced by Gaia will be affected by radiation damage causing charge loss and the deformation and shift of images. The agreed method to handle the radiation-damage effects at least for astrometry and photometry [1, 2] is through a ‘forward modelling’ approach, in which the observed photoelectron counts are compared with theoretical counts calculated by means of a suitable model. Adjusting the various model parameters until the best fit is obtained results in an estimation of the model parameters including those of the astronomical object (e.g., position and flux). The crucial point in this approach is that the radiation-damage effects are accounted for directly at the level of observed photoelectron counts, rather than further downstream in the data processing. It also avoids any attempt to derive ‘corrected’ photoelectron counts by some inversion process, which is intrinsically unstable and likely to introduce noise of unknown characteristics.

The general model of the photoelectron counts\(^1\) is outlined in Fig. 1. Successive steps of the model are:

- The scene is an idealized representation of the object, e.g., a scaled delta function on a constant background for a single star, or the sum of several scaled delta functions for a multiple star (the scaling factor represents the total flux of each component).

- The image is obtained by linear superposition of the corresponding (scaled and shifted) Point Spread Functions (PSFs) or Line Spread Functions (LSFs); mathematically, this is a convolution of the scene with the PSF/LSF.\(^2\) It is a continuous function expressed in expected number of photoelectron counts per pixel.

\(^1\)It is assumed that the raw CCD data, expressed in ADC units (LSB), can be converted to equivalent photoelectron counts by a known transformation involving bias, gain and non-linearity corrections. The resulting ‘counts’ are in general non-integer, and can be statistically modelled as a Poisson process with additional gaussian readout noise.

\(^2\)Actually, this operation is more general than a simple convolution, since the kernel (PSF/LSF) depends on additional properties of the scene, such as the spectrum of each point source. However, the important point is that the image can be described as a linear combination of such functions.
The sampled image is the set of intensity values in the discrete points representing the pixels.

The charge image is the sampled image modified by the CTI effects. The modification is described by the charge distortion model (CDM) and may depend on the detailed illumination history of the pixels in addition to the CDM parameters.

The modelled counts are obtained by binning the charge image, as required by the readout strategy.

The observed counts are statistically modelled as a random vector with expected values equal to the modelled counts, and known noise properties.

It should be noted that this model is already a very simplified picture of what goes on inside the CCD. It is also un-physical in that it completely ignores how the charge image is successively built up during the TDI, and how the charges are redistributed during this process. As a consequence, there is no physical entity in the real CCD corresponding to the ‘image’ (or ‘sampled image’) in Fig. 1; these are intermediate stages introduced purely for mathematical convenience.

2 Scope of the model

The purpose of this note is to define a first-order CDM with possible application in GIBIS and IDT/IDU. The particular model specified below (Sections 3–4) is denoted CDM-01. The parameter values mentioned in Sect. 5 are not part of the model. It is proposed that alternative models, or modifications of the present one, are sequentially numbered for easy reference.

Modelling of CTI effects can be made at many different levels of complexity and realism, and for different purposes [1]. The present model represents a phenomenological, macroscopic description of the various effects observed in test data, rather than a true physical model of what is really happening. It cannot replace more detailed, and physically much more realistic, models that are also being developed, for example using Monte Carlo techniques along the lines described in [3, 4]. However, for large-scale simulations, as well as for the actual treatment of raw data in the IDT and IDU, vastly simplified and computationally efficient models are needed. It is hoped that some model similar to the present CDM can be found that is sufficiently accurate for these purposes. In order to be practically feasible, the final model should at least not be vastly more complex than the present one.
3 Description of CDM-01

The input to the CDM is a sequence of values \( s_i \) representing the sampled image (Fig. 1) in successive TDI periods (or pixels) \( i \). The output is a sequence of distorted counts, \( d_i \), representing the charge image. The two sequences are related by

\[
d_i = s_i - c_i + r_i
\]

(1)

where \( c_i \) and \( r_i \) are, respectively, the number of electrons captured and released in the current step. The main part of the model is a specification of how \( c_i \) and \( r_i \) depend on current and previous pixel values:\(^3\)

\[
c_i = c(s_j, j \leq i)
\]

(2)

\[
r_i = r(s_j, j \leq i)
\]

(3)

However, instead of writing these explicitly as functions of an (in principle) infinite number of variables \( s_j \) \((j \leq i)\), we assume that they are functions of a small number of state variables \( f_i \), \( f_i^{(1)} \), \( f_i^{(2)} \), \( \ldots \) that are computed recursively. With \( f \equiv (f^{(0)}, f^{(1)}, f^{(2)}, \ldots) \) we thus have:

\[
c_i = c(s_i, f_i)
\]

(4)

\[
r_i = r(s_i, f_i)
\]

(5)

\[
f_i = f(s_i, f_{i-1})
\]

(6)

The present first-order model uses just a single state variable called the equivalent fill level and denoted \( f_i \) (dropping superscript in \( f^{(0)} \), or the vector notation for \( f_i \)). Roughly speaking, it should be understood as the minimum level of \( s_i \) required for part of the charges to be captured (i.e., a package with \( s_i < f_i \) would hardly see any empty traps).

3.1 Charge release model

Let \( N_{\text{filled}}(s_i, f_i) \) be the number of filled traps ‘seen’ by the current charge package. Assuming a single trap species with release time constant \( \tau \), the release function becomes

\[
r(s_i, f_i) = \left(1 - e^{-\Delta t/\tau} \right) N_{\text{filled}}(s_i, f_i) = \eta N_{\text{filled}}(s_i, f_i)
\]

(7)

where \( \Delta t \) is the TDI period and \( \eta = 1 - \exp(-\Delta t/\tau) \) for brevity. For \( \tau \gg \Delta t \) we have \( \eta \simeq \Delta t/\tau \).

For lack of more detailed information, we assume that \( N_{\text{filled}} \) is independent of \( s_i \) and proportional to \( f_i \); thus

\[
N_{\text{filled}}(s_i, f_i) = R f_i
\]

(8)

where \( R \geq 0 \) is a model parameter characterizing the degree of radiation damage, or equivalently the total number of traps. The release function is then simply

\[
r(s_i, f_i) = \eta R f_i
\]

(9)

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\(^3\)An alternative formulation would be to consider \( c \) and \( r \) to be functions of \((s_i, d_{i-1}, d_{i-2}, \ldots)\) rather than \((s_i, s_{i-1}, s_{i-2}, \ldots)\); since \( d_j \) for \( j < i \) depend on \( s_j \) through (1) the two formulations are clearly equivalent.
3.2 Charge capture model

The capture function \( c(s_i, f_i) \) should encapsulate the following behaviour: Let \( N_{\text{empty}} \) be the number of empty traps seen by the charge package, and \( N_c \) the number of charges available to fill them.

- In the limiting case when \( N_{\text{empty}} \ll N_c \), the number of captures is given by \( N_{\text{empty}} \).
- Conversely, when \( N_{\text{empty}} \gg N_c \), the number of captures is given by \( N_c \).
- When both \( N_{\text{empty}} \) and \( N_c \) are small, the number of captures is proportional to the product \( N_{\text{empty}}N_c \).

A simple function with this behaviour is

\[
c(s_i, f_i) = \frac{N_{\text{empty}}N_c}{N_{\text{empty}} + N_c + K}
\] (10)

where \( K \geq 0 \) is a parameter representing the inverse capture rate in the limiting case of small \( N_{\text{empty}} \) and \( N_c \).

Let \( F \) be the equivalent fill level corresponding to saturation of the traps (i.e., the value of \( f_i \) reached after passing a long series of big charge packages across the CCD). Then \( RF \) is the total number of traps and consequently

\[
N_{\text{empty}} = RF - N_{\text{filled}} = R(F - f_i)
\] (11)

In order to estimate the number of charges available to fill the traps we need to consider the different behaviours of the capture process at different signal sizes, resulting in different capture functions \( c_A, c_B, \) etc. A weighted mean of these functions, where the weights are determined by \( s_i \), then gives the effective capture function \( c \).

For small packages (subscript \( A \)), all the charges are available (\( N_c = s_i \)), but the probability of capture per charge is small because of the low charge density. Using (10) and (11) this gives

\[
c_A(s_i, f_i) = \frac{R(F - f_i)s_i}{R(F - f_i) + s_i + K_A}
\] (12)

where the parameter \( K_A \) may be adjusted to reflect the capture rate in this situation.

For large packages the effective fill level \( f_i \) acts as a threshold,

\[
N_c = \begin{cases} 
  s_i - f_i & \text{if } s_i > f_i \\
  0 & \text{otherwise}
\end{cases}
\] (13)

so that the capture function becomes

\[
c_B(s_i, f_i) = \begin{cases} 
  \frac{R(F - f_i)(s_i - f_i)}{R(F - f_i) + (s_i - f_i) + K_B} & \text{if } s_i > f_i \\
  0 & \text{otherwise}
\end{cases}
\] (14)

Finally we allow a smooth transition between the two cases by writing

\[
c(s_i, f_i) = q(s_i)c_A(s_i, f_i) + (1 - q(s_i))c_B(s_i, f_i)
\] (15)
where the transition function \( q(s) \) goes from 1 for small \( s \) to 0 for large. For the present model an exponential transition function is used,

\[
q(s_i) = \exp(-s_i/S)
\]

(16)

where the parameter \( S \) is the size of the charge package where the transition occurs. (Many other, perhaps more suitable transition functions could be envisaged.)

### 3.3 Propagation of the state variable

In the current TDI step \((i)\), the net number of charges captured is given by \( s_i - d_i = c_i - r_i \). Conservation of charges then requires that

\[
N_{\text{filled}}(s_{i+1}, f_{i+1}) = N_{\text{filled}}(s_i, f_i) + c_i - r_i
\]

(17)

Using (8) and (9) we get

\[
f_{i+1} = (1 - \eta)f_i + c_i/R
\]

(18)

which completes the model.

### 3.4 Initialization of the state variable

In principle \( f_i \) depends on the infinitely long prehistory of values \( s_j \) \((j \leq i)\). In practice, it can be initialized in two different ways:

1. Immediately after a sufficiently large charge injection, it can be assumed that \( f_i = F \).
2. After a sufficiently long, constant low-level illumination \( s_j = \beta \) (background), the trap filling reaches a constant level \( f_i = f_{ss} \) obtained by solving the steady-state equation

\[
c(\beta, f_{ss}) = r(\beta, f_{ss})
\]

(19)

If \( \beta \ll S \) and \( f_{ss} \ll F \), a useful approximate solution is

\[
f_{ss} = \frac{\beta}{(R + K_A/F)\eta}
\]

(20)

### 4 Algorithmic summary for CDM-01

General constants needed:
- \( \Delta t \) = the TDI period (0.9828 ms)

Model parameters:
- \( \tau \) = time constant for charge release
- \( F \) = equivalent fill level for trap saturation
- \( K_A \) = parameter representing the inverse capture rate for small signals
- \( K_B \) = parameter representing the inverse capture rate for large signals
- \( S \) = signal size for transition between small and large signals
- \( R \) = scaling parameter for the total number of traps

Input:
- \( \{s_i, i = 0, 1, \ldots\} \) = sampled image points (in photoelectrons)
Initialization:
\[ \eta = 1 - \exp(-\Delta t/\tau) \]
\[ f_0 = \text{initial value of the equivalent fill level}. \] This may be computed as in Sect. 3.4.

For \( i = 0, 1, 2, \ldots \) calculate:
- \( c(s_i, f_i) \) from (15), using (12), (14) and (16)
- \( r(s_i, f_i) \) from (9)
- \( d_i \) from (1)
- \( f_{i+1} \) from (18)

Output:
\[ \{d_i, i = 0, 1, \ldots \} = \text{charge image counts (in photoelectrons)} \]

5 A numerical example

In order to illustrate the effects of the above model, a series of images were passed through a model with the following parameter values:

\[ \tau = 100\Delta t, \quad F = 25000, \quad K_A = K_B = 25000, \quad S = 3000, \quad R = 0.5 \quad (21) \]

These particular values were chosen to give an overall visual impression of effects similar to what is observed in CCN10 test data; however, no attempt was made to adjust the parameters in order to reproduce the actual data.

Figures 2–5 show the results for a single gaussian LSF with flux \( \alpha = 10^5, 10^4, 10^3, \text{and } 10^2 \) e. The blue circles are the input counts \( (s_i) \), the red crosses the output counts \( (d_i) \). The input in this case was

\[ s_i = \beta + \frac{\alpha}{\sigma\sqrt{2\pi}} \exp \left( -\frac{(i - \kappa)^2}{2\sigma^2} \right) \quad (22) \]

with \( \beta = 10 \text{ e pix}^{-1}, \sigma = 1.05 \text{ pixel}, \text{and } \kappa = 10.5 \text{ pix}. \) However, in order to make the graphs look more continuous, they always show four different sequences, successively displaced by a quarter of a pixel. (The calculations are always stepped by whole pixels or TDI periods.) The initial value for the equivalent fill level was calculated from (20).

Figure 6 shows the response of the model to a charge injection shortly before the LSF, while Fig. ?? shows the response to a sequence of three equally bright, equidistant images. In this last figure, the equivalent fill level \( (f_i) \) for one of the sequences is also shown (continuous line).
Figure 2: Example of input (blue circles) and output (red crosses) of the CDM-01 model with parameters as in Sect. 5. This example is for a gaussian LSF with standard deviation 1.05 pixel and total flux $10^5$ photoelectrons.

Figure 3: Same as Fig. 2 but for a total flux of $10^4$ photoelectrons.
Figure 4: Same as Fig. 2 but for a total flux of $10^3$ photoelectrons.

Figure 5: Same as Fig. 2 but for a total flux of $10^2$ photoelectrons.
Figure 6: Example of input (blue circles) and output (red crosses) of the CDM-01 model with parameters as in Sect. 5. This example is for a gaussian LSF with standard deviation 1.05 pixel and total flux $10^4$ photoelectrons, preceded by charge injection ($2 \times 10^4$ electrons in four successive pixels).

Figure 7: Example of input (blue circles) and output (red crosses) of the CDM-01 model with parameters as in Sect. 5. This example is for a ‘triple star’ of gaussian LSFs, each with standard deviation 1.05 pixel and a total flux of $5 \times 10^4$ photoelectrons. The blue line shows the equivalent fill level ($f_i$).
6 Possible extensions and improvements of the model

There are several obvious ways in which the above model could be extended to provide increased flexibility (thus better representing the test data), or otherwise improved. For example:

1. The charge release model (7) implicitly assumes exponential decay with a single time constant, while test data clearly demonstrate the presence of several different time constants (interpreted as different trap species). This can be modelled by superposing different model components (one per time constant or species). Some analysis is needed to find out which of the other model parameters should be common for all the components, and which should be separately adjustable.

2. The charge capture model (15), being a mix of two different functions, does not look very convincing. It could be replaced by a single function, valid over the full range of $s_i$, and perhaps better founded in the physics of the capture process.

3. The relation between the state variable $f_i$ and the number of filled traps could be more general that the strict proportionality in (8).

4. It is possible (and probably necessary) to use more than a single dimension of the state variable $f_i$. Perhaps they should represent the trap filling state at different signal levels (e.g., small, medium, large).

References


