PSF and LSF representation
for the simulation of Gaia-3 Astro data

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Abstract. This note describes a proposed representation of the general polychromatic line spread functions (LSF) and point spread functions (PSF) of the Gaia-3 Astro instrument for use in simulations.

1 Introduction

The calculation of LSF and PSF to various levels of detail has been discussed e.g. in [1, 2, 4, 5]. The description given here extends the model in [4] to a form suitable for efficient simulation of Astro data for Gaia-3. It is based on the following main ideas:

- the polychromatic LSF and PSF are flux-weighted linear combinations of a small (~ 5) number of quasi-monochromatic LSF and PSF [1, 5];
- the quasi-monochromatic PSF are the products of the corresponding AL and AC quasi-monochromatic LSF [2];
- the quasi-monochromatic LSF (AL or AC) is expressed as the sum of a Lorentz profile, representing the diffraction wings, and a bi-quartic spline representing the core of the image [2, 4].

For convenience, some formulae from the previous references are repeated below. In order to facilitate cross-referencing to the corresponding Java code, the indexing convention has been changed so that all indices start at 0.

2 Calculation of LSF

2.1 Monochromatic LSF

Let $u$ be the AL coordinate measured in AL pixels, and $v$ the AC coordinate measured in AC pixels. The origin $(u, v) = (0, 0)$ corresponds to the image centroid in the geometrical optics limit (see Sect. 2.2). The monochromatic LSF $L_\lambda(u)$ (AL) and $C_\lambda(v)$ (AC) are calculated by taking into account:
1. diffraction, including a specified set of WFE maps expressed in terms of normalized Legendre polynomials (see Sect. 2.2);

2. averaging over the pixel area (convolution with a rectangular function of unit width in \(u\) or \(v\));

3. in the AL case, motion of the optical image during one TDI phase (convolution with a rectangular function of width \(1/n_{\text{phase}}\) in \(u\), where \(n_{\text{phase}} = 4\));\(^1\)

4. in the AC case, motion of the optical image during one CCD crossing (convolution with a rectangular function of width 2.75 pixels in \(v\) – this is the mean transverse motion, equal to \(2/\pi\) times the maximum value);

5. the combined image smearing from other effects such as AOCS errors, differential distortion, and charge diffusion, is modelled as a Gaussian with standard deviation 4.2 \(\mu\)m (the major part, 4.0 \(\mu\)m, is due to charge diffusion).

The smearing due to pixel size, image motion and other effects (except aberrations) is symmetric in respective \(u\) or \(v\). In these calculations, the discretization of the pupil etc should be such that aliasing effects are less than 1% within the \(\pm 10\) pixel core of the computed LSF.

The resulting monochromatic LSF \(L_\lambda(u)\) and \(C_\lambda(v)\) are normalized to unit area when the coordinates are expressed in pixels,

\[
\int_{-\infty}^{+\infty} L_\lambda(u) \, du = 1, \quad \int_{-\infty}^{+\infty} C_\lambda(v) \, dv = 1
\]

(1)

2.2 Assumed wavefront errors

The WFE at any point of the Astro field is represented by a set of coefficients \(Q_{ij}\) such that

\[
w(x, y) = \sum_{i+j>0} Q_{ij} N_i(2x/D) N_j(2y/H)
\]

(2)

where \((x, y)\) are (AL,AC) linear coordinates in the pupil, \(D \times H\) are the dimensions of the pupil, and \(N_n(z) = (2n + 1)^{1/2} L_n(z)\) are normalized Legendre polynomials [5].

Note that (2) does not contain any tilt terms, implying \(Q_{01} = Q_{10} = 0\); this defines the origin of \((u, v)\) to be at the long-wavelength (or ‘geometrical optics’) limit of the image centroid [3].

2.3 Quasi-monochromatic LSF

Quasi-monochromatic functions \(L_k(u)\) and \(C_k(v)\) are obtained for \(n_{\text{band}} = 5\) wavelength bands \((k = 0 \ldots n_{\text{band}} - 1)\) as weighted means of the corresponding monochromatic functions, using the ‘basic spectra’ \(B_k(\lambda)\) as weight function [1]. The basic spectra are uniquely

\(^1\)In reality the four phases do not have the same size, since the 10 \(\mu\)m pixel pitch is divided into \(2 + 3 + 2 + 3 \mu\)m steps. The resulting effect on the LSF can in practice be ignored at least for the present simulations.
defined by the knot sequence \( (\lambda_0, \lambda_1, \ldots, \lambda_{n_{\text{band}}+1}) = (330, 375, 430, 500, 600, 750, 1000) \text{nm} \) \(^{(3)}\)

through

\[
B_k(\lambda) = \frac{2}{\lambda_{k+2} - \lambda_k} \times \begin{cases} 
0 & \text{if } \lambda < \lambda_k \\
(\lambda - \lambda_k)/(\lambda_{k+1} - \lambda_k) & \text{if } \lambda_k \leq \lambda < \lambda_{k+1} \\
(\lambda_{k+2} - \lambda)/(\lambda_{k+2} - \lambda_{k+1}) & \text{if } \lambda_{k+1} \leq \lambda < \lambda_{k+2} \\
0 & \text{if } \lambda_{k+2} \leq \lambda 
\end{cases} \tag{4}\]

3 Representation of LSF and PSF

This section describes the representations in which the LSF and PSF shall be given and used by the simulations.

3.1 Quasi-monochromatic LSF

Each quasi-monochromatic LSF \( L_k(u) \) and \( C_k(v) \) (for \( k = 0 \ldots n_{\text{band}} - 1 \)) is represented as a sum of a Lorentz profile (for the wings) and a bi-quartic spline (for the core); see [2]. The knot sequence for the spline is

\[
u_i = u_0 + i\delta u, \quad i = 0 \ldots n - 1
\tag{5}\]

where \( n \) is the number of spline coefficients and \( \delta u \) the knot separation (grid spacing). The following parameters will be used:

1. grid spacing \( \delta u = \delta v = 0.5 \);
2. number of bi-quartic spline coefficients \( n = 31 \) (TBC);
3. coordinate origin \( u_0, v_0 \) chosen such that the central knot of the spline is at \( u = v = 0 \) (the adopted image centroid), which gives \( u_0 = -\frac{1}{2}(n - 1)\delta u \) and \( v_0 = -\frac{1}{2}(n - 1)\delta v \) in all bands;\(^3\)
4. fraction of the intensity in the wings, \( h = 0.1 \) (TBC).

The adopted fraction \( h \) determines the parameters \( a_0 \) and \( a_2 \) according to [4]

\[
a_0 = \frac{2D_{\text{SL}}}{f\lambda_{k+1}} h^2, \quad a_2 = \frac{h}{\pi a_0}
\tag{6}\]

\(^2\)The given sequence is just a subdivision of the total \( G \) band [330, 1000] nm that is roughly equidistant in wavenumber. An improved choice could take into account the known kinks in the effective spectrum due to, for example, the Balmer jump and the Ag reflectivity below 400 nm.

\(^3\)Note that this is a major difference with respect to [4], where the LSF was separately centred for each wavelength band and then displaced according to given chromaticity coefficients \( \Gamma_k \) when computing the polychromatic LSF.
with similar expressions for AC, using $Hs_{AC}$ instead of $Ds_{AL}$. Here $D$ and $H$ are the pupil dimensions AL and AC, $s_{AL}$ and $s_{AC}$ the linear pixel sizes AL and AC, $f$ the effective focal length, and $\lambda_{k+1}$ the central wavelength for band $k$.

For the items above marked TBC it should be verified that the proposed choice provides reasonable accuracy of the representation.

### 3.2 Quasi-monochromatic PSF

Quasi-monochromatic PSF are represented as [2]

$$P_k(u, v) = L_k(u)C_k(v)$$  \hfill (7)

The averaging over the basic spectra preserves the areas of the quasi-monochromatic LSF and PSF so that they satisfy

$$\int_{-\infty}^{+\infty} L_k(u) \, du = 1, \quad \int_{-\infty}^{+\infty} C_k(v) \, dv = 1, \quad \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P_k(u, v) \, du \, dv = 1$$  \hfill (8)

### 3.3 Polychromatic LSF and PSF

For an arbitrary spectrum, polychromatic LSF and PSF are calculated as linear combinations of the $n_{\text{band}} = 5$ quasi-monochromatic functions. For example, the AL LSF is

$$L(u|\text{spectrum}) = \sum_{k=0}^{n_{\text{band}}-1} s_k L_k(u)$$  \hfill (9)

where $s_k \geq 0$ are coefficients depending on the effective spectrum (i.e., including instrument transmittance and CCD quantum efficiency) and normalized to $\sum_k s_k = 1$. A similar expression applies to the AC LSF $C(v)$ and to the PSF $P(u, v)$.

### 4 Requirements for the first set of LSF

The first set of LSF to be used in Gaia-3 simulations will not include any effects related to chromaticity or field-dependent LSF variations. Thus they should satisfy the following constraints:

1. only one WFE map (one set of coefficients $Q_{ij}$) shall be used for the whole Astro field and in both viewing directions;
2. the WFE map should only include even aberrations, thus making both the AL and AC LSF completely symmetric about $u = v = 0$;
3. nevertheless, to have a realistic LSF degradation, the total RMS WFE (in the low-frequency part represented by Eq. 2) should be 40 nm. For example, one could use $Q_{02} = Q_{20} = Q_{04} = Q_{40} = 20$ nm (TBC).
Acknowledgment

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References


Acronyms

The following table has been generated from the on-line Gaia acronym list:

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<td>AL</td>
<td>ALong scan (direction)</td>
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<td>Attitude and Orbit Control Sub-system</td>
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<td>CCD</td>
<td>Charge-Coupled Device</td>
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