Multi-pass scanning across Baade’s Window

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Abstract. In order to observe high-density regions like Baade’s Window (BW) without excessive demands on the on-board data handling it may be desirable to modify the scanning law to give more scans across such regions. A modified scanning law (MSL) that allows this has been derived and compared with the performance of the nominal scanning law (NSL). Assuming that stars in BW are observed with 25% probability each time they enter one of the Astro fields, the NSL results in an increase of the astrometric errors by a factor 2–3, while the MSL limits this factor to 1.2–1.4. The MSL is achieved without significantly affecting the performance in other areas of the sky or increasing the maximum transverse scan rate. However, if several areas like BW need to be scanned in a similar fashion, it will be necessary to increase the maximum allowed transverse scan rate in other areas of the sky in order to maintain good sky uniformity.

1 Introduction

At a limiting magnitude of $G = 20$, the maximum (smoothed) star density encountered by Gaia outside of globular clusters is assumed to be about $3 \times 10^6 \, \text{deg}^{-2}$. One of Baade’s Windows, specifically the one centred on the globular cluster NGC6522 at galactic coordinates $\ell = 1.04^\circ$, $b = -3.88^\circ$, can be taken as representative for a handful of such extreme areas. The window, hereafter called BW, is irregular in shape but roughly circular, with a diameter slightly less than $1^\circ$. Sizing the CCD windowing and on-board data handling to cope with these very few extreme sky areas may be difficult and costly. It is therefore of great interest to investigate if a lower maximum density, say $0.75 \times 10^6 \, \text{deg}^{-2}$ or 25% of the expected maximum value, might be acceptable.

Imposing a lower maximum density than actually encountered implies a real-time selection of the stars to be observed in these areas, either by default (e.g., stop observing when the maximum has been reached) or by design (e.g., changing the detection threshold or making an active selection by some other criterion). The various strategies for this selection are not discussed here. I will simply assume that an effectively random selection with specified probability $p$ can be implemented, so that on average only one out of $p^{-1}$ detected objects is selected for further observation across the Astro field of view. ($p = 0.25$ is assumed below.) We may call this diluted scanning, as opposed to the normally assumed compact scanning, where an object is observed at every possible opportunity.

If the nominal scanning law (NSL) is used, such diluted scanning of high-density regions means that the total number of transits, as well as the total observing time on any object,
The open circles show a configuration of 19 test points in BW, covering an area of 1° diameter centred on \( \lambda = 270.75^\circ, \beta = -6.56^\circ \). In the diluted scanning, these points are observed with probability 0.25, while the crosses (just outside BW) are observed with probability 1.

is reduced by the factor \( p \). Statistically, this will increase the astrometric standard errors at least by a factor \( p^{-1/2} \), but the effect may be much worse on specific objects because of small-number statistics.

In order to mitigate these effects it has been suggested that the scanning law could be modified to provide an increased number of scans across BW at the relevant epochs, so-called multi-pass scanning. For example, if the number of scans across BW is increased by a factor \( p^{-1} \), then this should statistically compensate for the diluted scanning and reduce fluctuations from small-number statistics.

The purpose of this note is to examine the feasibility of this idea by (i) designing a modified scanning law (MSL) that provides multi-pass scanning across BW; (ii) estimating the resulting loss/gain in astrometric accuracy for objects inside BW; (iii) investigate the possible negative effects for other objects over the sky, and thus on the overall mission accuracy.

2 Assumptions

Gaia-2 parameters are assumed, in particular a solar aspect angle of \( \xi = 50^\circ \), 5.2 revolutions per year over 5 year mission, a transverse Astro FOV of 0.74°, and scan rate 60 arcsec s\(^{-1}\). For simplicity no dead time is assumed, nor any dead zones between the CCDs in the across-scan direction.

With these assumptions, the mean number of Astro transits for an arbitrary point on the sky is

\[
(2 \text{ FOVs}) \times \sin(0.37^\circ) \times (4 \text{ spin/day}) \times 365.25 \times 5 = 94.35
\]

This number is independent of the scanning law (for a given spin rate), as long as an object
is observed on every possible transit, and is therefore a useful check of the simulations both for the nominal and modified scanning law.

For the present study BW is assumed to be circular, with a sharp boundary and a diameter of 1°, and centred on

\[
\begin{align*}
\ell &= 1.04^\circ, \quad b = -3.88^\circ \quad \text{(galactic coordinates)} \\
\alpha &= 270.86^\circ, \quad \delta = -30.00^\circ \quad \text{(equatorial coordinates)} \\
\lambda &= 270.75^\circ, \quad \beta = -6.56^\circ \quad \text{(ecliptic coordinates)} \\
\end{align*}
\]

(2)

In the following, ecliptic coordinates are used throughout. 19 test points were placed within a radius of 0.5° of the BW centre, as shown by the small open circles in Fig. 1. In the diluted scanning, these 19 points are observed with probability \( p = 0.25 \), while all other points on the sky (including the crosses in Fig. 1) are assumed to be observed with probability 1.

### 3 Scanning laws

Details about the assumed scanning laws are given in the Appendix.

The scanning law for Gaia is constrained by (i) the fixed angle \( \xi \) between the Sun and the spin (z) axis; (ii) the maximum speed of the z axis on the sky, which determines the maximum transverse scan rate during any field of view transit; (iii) the fixed inertial spin rate \( \omega_z \) about the z axis. The speed of the z axis on the sky is related to the speed of the Sun by the ratio \( S = |\dot{z}| \lambda_s^{-1} \), where \( \lambda_s \) is the ecliptic longitude of the (nominal) Sun. Together with a specification of the sense of revolution of the spin axis about the solar direction, these conditions result in two differential equations for the heliotropic angles \( \nu \) and \( \Omega \) as functions of time (see Eq. 10 and 5 in the Appendix). The function \( \nu(t) \) describes how the z axis revolves around the Sun, while \( \Omega(t) \) describes how the satellite spins around z.

In addition to the constrains (i)–(iii) above, there is another (somewhat less stringent) requirement on sky uniformity, namely (iv) that the mean revolving rate should be at least \( \langle \dot{\nu} \rangle \simeq (260^\circ/\xi) \dot{\lambda}_s \).

In the current nominal scanning law (NSL), \( \xi = 50^\circ, \ S \equiv S_N = 4.095, \ \omega_z = 60 \text{ arcsec s}^{-1}, \) and the sense of revolution is positive (\( \dot{\nu} > 0 \)). The fixed \( S \) means that \( \nu(t) \) is increasing at the maximum rate compatible with the constraint on the transverse scan rate.

The only way to modify the scan law and still comply with the ‘hard’ constraints (i)–(iii) is to allow \( S \) to occasionally become smaller than the maximum (nominal) value \( S_N \). However, this cannot happen too often, or too long, because that will violate (iv).

The Modified Scanning Law (MSL) described in A.3 agrees with the NSL when the scanning plane (normal to z) is well away from BW, i.e., the spin axis moves across the sky at the constant rate \( S_N \). However, as soon as the scanning plane is within half the transverse field of view from any part of BW, the speed of the z axis is reduced (if necessary) such that the transverse speed of BW is at most 25% of the maximum speed allowed by the
NSL. In this way, at least 4 successive scans of the BW area is guaranteed (whereas only one scan is guaranteed by the NSL).

The effect is illustrated in Fig. 2, which shows the motion of the spin axis during the first and second years. The dashed curve shows the path of $z$ according to the NSL, the solid curve shows the MSL. The dotted curve is a great circle with a pole at the centre of BW. Clearly BW can only be observed when the $z$ axis is within $\sim 1^\circ$ of this great circle.

In the MSL there is effectively a ‘speed limit’ on the transverse scan rate as the scanning plane moves across BW. This means that $\nu(t)$ lags behind the nominal law $\nu_N(t)$ more and more after each crossing over BW. After 5 years ($\nu_N \simeq 26 \cdot 360^\circ$) the accumulated lag is $\nu_N - \nu = 182^\circ$. This is still (just) compatible with the constraint (iv), and thus acceptable in view of the sky uniformity (as will be shown below).

It can be noted that the maximum across-scan rate cannot be reduced to less than about 25% of the nominal value. A smaller value might require that the speed of the spin axis becomes less than that of the Sun, which is (in general) unfeasible.

4 Results of the sky scanning

Observations of the test points in Fig. 1 were simulated in the following three cases:

1. Reference case (NSL with $p = 1$): the NSL was used and the test points were observed on every possible scan;

2. Diluted nominal case (NSL with $p = 0.25$): the NSL was used, but the 19 test points inside BW were observed with probability 0.25 per FOV transit;

3. Diluted modified case (MSL with $p = 0.25$): the MSL was used, and the test points inside BW were observed with probability 0.25 per FOV transit.

Figure 3 shows the temporal distribution of the observations in the three cases.

Table 1 gives statistics for the number of FOV transits ($n$) and the coefficients of improvement $C$ for the five astrometric parameters in ecliptic coordinates. The table gives mean values and 90th percentiles, the latter defined in the sense that 90% of the stars obtained better results (more transits or smaller coefficients of improvement) than the given values. Because of the statistical scatter caused by the random selection in Case 2 and 3, the 90th percentiles may be a better indication of the overall quality than mean values.

The results in Table 1 should be compared with the average performance on the part of the sky outside of BW. To this end 5000 points were randomly scattered over the celestial sphere and observed with the NSL and MSL (always with $p = 1$). In this case there is no difference between Case 1 and 2. Results are given in Table 2.

Figure 4 shows the coefficient of improvement in parallax ($C_\pi$) versus ecliptic latitude in the three cases. Results for the 5000 random points are shown as dots, the 19 test points inside BW as open circles, and 18 test points just outside BW (Fig. 1) as crosses.
Figure 2: **Top** – Motion of the spin (\(z\)) axis on the sky, in ecliptic coordinates, for the nominal scanning law (dashed curve) and modified scanning law (solid curve), during the first \(\simeq 10\) months of scanning. The dotted curve is a great circle having its pole at BW (indicated by a small cross near longitude 270°). **Middle** – Same but for the second year of scanning. Note how the MSL (solid curve) progressively lags behind the NSL (dashed curve). **Bottom** – Magnification of a part of the second-year map.
Figure 3: Distribution of observation epochs for the 19 test points in BW in the three cases of scanning described in the text. **Top** – Case 1 (NSL with $p = 1$). **Middle** – Case 2 (NSL with $p = 0.25$). **Bottom** – Case 3 (MSL with $p = 0.25$).
Table 1: Results for the 19 test points in BW for the three cases of scanning described in the text. \( n \) = number of Astro field transits; \( C \) = coefficient of improvement for the different astrometric parameters.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Case 1</th>
<th></th>
<th>Case 2</th>
<th></th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>90%</td>
<td>Mean</td>
<td>90%</td>
<td>Mean</td>
</tr>
<tr>
<td>( n )</td>
<td>74.2</td>
<td>70.0</td>
<td>18.5</td>
<td>13.0</td>
<td>49.5</td>
</tr>
<tr>
<td>( C_{\lambda} )</td>
<td>0.206</td>
<td>0.213</td>
<td>0.507</td>
<td>0.572</td>
<td>0.243</td>
</tr>
<tr>
<td>( C_{\beta} )</td>
<td>0.148</td>
<td>0.153</td>
<td>0.332</td>
<td>0.381</td>
<td>0.199</td>
</tr>
<tr>
<td>( C_{\pi} )</td>
<td>0.232</td>
<td>0.239</td>
<td>0.539</td>
<td>0.616</td>
<td>0.274</td>
</tr>
<tr>
<td>( C_{\mu, \lambda} )</td>
<td>0.149</td>
<td>0.153</td>
<td>0.290</td>
<td>0.462</td>
<td>0.177</td>
</tr>
<tr>
<td>( C_{\mu, \beta} )</td>
<td>0.118</td>
<td>0.121</td>
<td>0.282</td>
<td>0.362</td>
<td>0.139</td>
</tr>
</tbody>
</table>

Table 2: Results for 5000 test points randomly scattered over the sky and observed at every opportunity given by the NSL (Case 1 & 2) and MSL (Case 3).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Case 1 &amp; 2</th>
<th></th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>90%</td>
<td>Mean</td>
</tr>
<tr>
<td>( n )</td>
<td>94.6</td>
<td>68.0</td>
<td>94.2</td>
</tr>
<tr>
<td>( C_{\lambda} )</td>
<td>0.163</td>
<td>0.207</td>
<td>0.166</td>
</tr>
<tr>
<td>( C_{\beta} )</td>
<td>0.152</td>
<td>0.169</td>
<td>0.156</td>
</tr>
<tr>
<td>( C_{\pi} )</td>
<td>0.199</td>
<td>0.235</td>
<td>0.200</td>
</tr>
<tr>
<td>( C_{\mu, \lambda} )</td>
<td>0.115</td>
<td>0.147</td>
<td>0.116</td>
</tr>
<tr>
<td>( C_{\mu, \beta} )</td>
<td>0.107</td>
<td>0.121</td>
<td>0.109</td>
</tr>
</tbody>
</table>

5 Conclusions

A modified scanning law (MSL) has been specified, respecting all the important constraints (constant solar aspect angle, no increase in the maximum transverse scan rate, good sky uniformity) but giving a significantly increased number of scans across BW. This allows the objects in BW to be observed only in a certain fraction \( p \) of the field transits, thus reducing the maximum star density that must be handled on-board.

Comparing Case 1 and 3 in Table 1, we see that if BW is observed with MSL at \( p = 0.25 \), the astrometric errors are increased by about 20% on average, and 30–40% in the worst cases (90th percentiles). This is achieved without significantly affecting the performance in other areas of the sky (Table 2) or increasing the maximum transverse scan rate.

However, if several areas like BW need to be scanned in a similar fashion, it will be necessary to increase the transverse scan rate in other areas of the sky in order to maintain good sky uniformity.
Figure 4: Distribution of the coefficient of improvement in parallax ($C_\pi$) as function of ecliptic latitude. **Top** – Case 1 (compact NSL), showing only the 5000 random points on the sky. **Middle** – Case 2 (diluted NSL), including also the 19 test points inside BW (open circles) that are observed with probability $p = 0.25$. **Bottom** – Case 3 (diluted MSL), with the 18 test points just outside BW (observed with probability $p = 1$) marked as crosses. The dashed horizontal line marks $C_\pi = 0.25$. Note difference in scale!
Appendix: Details on the scanning laws

A.1. General properties of feasible scanning laws

Some useful results from GAIA–LL–035 are repeated here for convenience.

Let $k$, $s$, and $z$ be unit vectors towards the north ecliptic pole, the (nominal) Sun, and the (positive) spin axis of Gaia. The longitude of the (nominal) Sun is $\lambda_s$. The scanning law must always be such that the solar aspect angle $\xi$ between $s$ and $z$ remains fixed. The revolving motion of $z$ about $s$ relative to the ecliptic plane is described by the revolving phase $\nu(t)$. The spin about $z$ is described by the spin phase $\Omega(t)$ of the $x$ axis (bisecting the two Astro viewing directions) relative to the plane containing $z$ and $s$.

The total inertial rotation of the satellite is made up of three rotations: 1. by the rate $\dot{\lambda}_s$ about $k$; 2. by the rate $\dot{\nu}$ about $s$; and 3. by the rate $\dot{\Omega}$ about $z$; thus,

$$\mathbf{\omega} = k\dot{\lambda}_s + s\dot{\nu} + z\dot{\Omega}. \tag{3}$$

The rate of change of the spin axis direction is

$$\dot{z} = \mathbf{\omega} \times z = (k \times z)\dot{\lambda}_s + (s \times z)\dot{\nu}. \tag{4}$$

The spin rate about $z$ is

$$\omega_z = \mathbf{\omega}' z = k' z \dot{\lambda}_s + s' z \dot{\nu} + \dot{\Omega} = \lambda_s \sin\xi \sin\nu + \dot{\nu} \cos\xi + \dot{\Omega}. \tag{5}$$

For operational reasons, the along-scan spin rate $\omega_z$ must also remain fixed. Thus, the only function that we can play with in order to modify the scanning law is $\nu(t)$. Once $\nu(t)$ has been determined, $\Omega(t)$ follows by integration of $\dot{\Omega}$ from (5). Additional constraints on $\nu(t)$ follow from the maximum across-scan rate and the condition for good sky coverage (cf. SAG–LL–014).

Rates may be expressed as the dimensionless quantities

$$S \equiv |\dot{z}| \lambda_s^{-1}; \quad \tag{6}$$

$$s \equiv \dot{z} \lambda_s^{-1}; \quad \tag{7}$$

$$\kappa \equiv \dot{\nu} \lambda_s^{-1}. \quad \tag{8}$$

If $c$ is the unit vector towards an object, its across-scan coordinate is given by $z = z'c$ and its across-scan rate by $\dot{z} = \dot{z}'c$ (assuming that $\dot{c} = 0$); hence

$$s = (k \times z)'c + (s \times z)'c \kappa = s_0 + s_1 \kappa, \tag{9}$$

where $s_0$ and $s_1$ are known constants for any transit of the object at $c$. The across-scan rate is bounded by $|\dot{z}| \leq |\dot{z}|$, or $|s| \leq S$.

5.1 The nominal scanning law (NSL)

The nominal scanning law (distinguished with subscript $N$ where necessary) is defined by a fixed $S = S_N$. $\nu_N(t)$ follows by integrating $\dot{\nu}_N = \dot{\lambda}_s \kappa_N$, where

$$\kappa_N = \sqrt{\frac{S_N^2 - \cos^2 \nu_N + \cos\xi \sin\nu_N}{\sin\xi}} \tag{10}$$
Ω_N(t) then follows from (5) with \( \omega_z = 60 \text{ arcsec s}^{-1} \).

Using \( \xi = 50^\circ \), \( S_N = 4.095 \), and \( \nu_N(0) = 0 \), a table of \( z_N(t) \) and \( \Omega_N(t) \) over 5 years was created by numerical integration, using a step length of 1/32 day (about 45° increment in \( \Omega_N \)). This was used to simulate the transits in Case 1 and 2.

Numerical integration of the scanning law was used rather than the explicit analytical form in GAIA–FM–010 and GAIA–FM–017, because the differential equation for \( \kappa \) is readily adapted to the modification introduced below.

The value \( S_N = 4.095 \) was selected to give, for \( \xi = 50^\circ \), a suitable sky coverage and a total of 26 complete revolutions of the spin axis about the solar direction in 5 years, or \( \nu_N(5 \text{ year}) = 9360^\circ \).

5.2 The modified scanning law (MSL)

The nominal scanning law just about guarantees one scan across an arbitrary object for every potential epoch (when the object crosses the scanning plane normal to \( z \)).

We now want to modify the scanning law so as to guarantee at least \( p^{-1} \) scans across the direction \( c \) at every potential epoch. We do this by imposing a much stricter constraint on the across-scan rate \( s \) whenever \( z'|c| \simeq 0 \): instead of the constraint \( |s| \leq S_N \) valid for the normal scanning (well away from \( c \)), we require

\[
|s| \leq pS_N
\]

where \( p = 0.25 \). We can think of this as a ‘speed limit’ for the motion of the scanning plane across \( c \). Since \( \kappa_N > 0 \), and we want to modify the scanning as little as possible, we seek the largest \( \kappa \leq \kappa_N \) satisfying

\[
|s_0 + s_1\kappa| \leq qS_N
\]

The solution is:

\[
\kappa = \begin{cases} 
\kappa_N & \text{if } s_1 = 0 \\
\min(\kappa_N, \frac{pS_N - s_0}{s_1}) & \text{if } s_1 > 0 \\
\min(\kappa_N, \frac{pS_N + s_0}{-s_1}) & \text{if } s_1 < 0 
\end{cases}
\]

This value of \( \kappa \) is used instead of \( \kappa_N \) from (10) whenever a part of BW is within half the field width from the scanning plane, i.e., when

\[
|z'|c| \leq \frac{1}{2}(\Phi + D)
\]

where \( \Phi = 0.74^\circ \) is the transverse field of view and \( D = 1^\circ \) the diameter of BW. The integration of the MSL only requires modification of the subroutine for computing \( \dot{\nu} \) as function of \( \dot{\lambda}_s \), \( s \), \( c \) and \( \nu \). A table of \( \nu(t) \) and \( \Omega(t) \) was thus constructed as for the NSL.