Use of parallax information in the Photometric System design

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Abstract. I propose that the figure of merit (FoM) for PS optimization is modified to take into account parallax information. This can be done simply by using an a priori assumption about the stellar mass, including its uncertainty. The required procedure is described.

In GAIA–LL–047 a procedure was proposed for calculating a figure of merit (FoM) for any proposed Gaia photometric system (PS). The FoM is based on the estimated precision by which the adopted set of astrophysical parameters (AP) can be jointly determined. This procedure was subsequently implemented by A. Brown (PWG–AB–003) and will be used to arrive at a recommendation for the choice of PS.

In the present implementation, the FoM does not take into account that parallaxes in many cases could help to determine the AP, in particular log \( g \) (strongly correlated with luminosity). GAIA–LL–047 described the principle for introducing parallax information into the FoM by means of an observation equation (Eq. 10 in GAIA–LL–047) relating the observed parallax \( \pi_{\text{obs}} \) to the value \( \pi(p) \) computed from the AP (contained in the vector \( p \)), and the observed magnitude \( V \).\(^1\) The result is that the quantities \( b_k b_k' \) should be added to the a priori information matrix \( B \), where:

\[
b_k = (0.2 \ln 10) \frac{\pi(p)}{\sigma_\pi} \frac{\partial M}{\partial p_k}
\]

Here, \( M = M_V + A_V \).

The PS design currently considers the following set of AP: \( A_V, \log{T_{\text{eff}}}, \log{g}, \) and \([\text{Fe/H}]\) (perhaps also \([\alpha/\text{Fe}]\)). A problem when implementing (1) is that this particular set of AP does not uniquely determine the luminosity of the star (and therefore not \( M_V \) and \( M \)), and it is consequently not clear how to compute the derivatives in (1).

With usual notations (in particular \( M \) for the mass), the fundamental relations relevant to the problem are:

\[
\begin{align*}
g & \propto M/R^2 \\
L & \propto R^2 T_{\text{eff}}^4 \\
M_{\text{bol}} & = \text{const} - 2.5 \log{L} \\
M_V & = M_{\text{bol}} - BC
\end{align*}
\]

\(^1\)Unlike in GAIA–LL-047, we use here \( V \) rather than \( G \) for consistency with the AP \( A_V \). It is assumed that the transformation from Gaia photometry to \( V \) is accurate compared with other error sources.
Combining these relations we have for the \( M = M_V + A_V \) in (1):

\[
M = \text{const} + A_V - 10 \log T_{\text{eff}} + 2.5 \log g - BC(p) - 2.5 \log M
\]  

(3)

from which

\[
\begin{align*}
\frac{\partial M}{\partial A_V} &= 1 \\
\frac{\partial M}{\partial \log T_{\text{eff}}} &= -10 - \frac{\partial BC}{\partial \log T_{\text{eff}}} \\
\frac{\partial M}{\partial \log g} &= 2.5 - \frac{\partial BC}{\partial \log g} \\
\frac{\partial M}{\partial [\text{Fe/H}]} &= -\frac{\partial BC}{\partial [\text{Fe/H}]} \\
\end{align*}
\]  

(4)

The partial derivatives of the bolometric correction can be derived from the SED library. The variation with luminosity class and (perhaps) metallicity is rather small, but \( \partial BC/\partial \log T_{\text{eff}} \) is significant (of order \( \pm 10 \)) except in a small temperature range (Fig. 1).

In (3) we have the parameter \( \log M \) on the right-hand side, which is not one of the AP considered, nor is it a unique function of the AP. This term therefore weakens the information contributed by the parallax, but fortunately not so much as to invalidate the process. The argument here is that the main variation in \( g \), e.g. between dwarfs and giants, is not caused by \( M \) but by \( R \) or \( L \) (since \( g \propto MT_{\text{eff}}^2 / L \)).
For a given set of AP one can make a certain a priori prediction of \( \log M \), including its uncertainty \( \epsilon \). For main-sequence stars the prediction might be reasonably precise (\( \epsilon \sim 0.05 \text{dex} ? \)) thanks to the mass-luminosity relation, but for giants the uncertainty is much larger (\( \sim 0.3 \text{dex} ? \)). Let us assume, for simplicity, that a constant value (say \( \epsilon = 0.3 \)) applies globally. The effect of the uncertainty in \( \log M \) is similar to that of a relative error in the parallax, and can be formally introduced in the observation equation by quadratically adding \( \epsilon \) (properly scaled) to the relative parallax error. The result is the following modified version of Eq. (1):

\[
 b_k = (0.2 \ln 10) \left[ \left( \frac{\sigma_\pi}{\pi} \right)^2 + (0.5 \ln 10)^2 \epsilon^2 \right]^{-1/2} \frac{\partial M}{\partial p_k} 
\]

(5)

How much the parallax information will contribute to the determination of \( \log g \) depends on both \( \sigma_\pi / \pi \) and \( \epsilon \). When \( \sigma_\pi / \pi \gg 1 \), all \( b_k \) become small and effectively nothing is added to the a priori information matrix. With decreasing \( \sigma_\pi / \pi \), more information is added, until \( \sigma_\pi / \pi \simeq (0.5 \ln 10) \epsilon \simeq 0.35 \), at which point the a priori uncertainty of the mass will become the limiting factor. In the limiting case when \( A_V \) and \( \log T_{\text{eff}} \) are well-determined but \( \log g \) not at all determined photometrically, then a good parallax will give \( \log g \) to a precision equal to \( \epsilon \).

I propose that the FoM is calculated with (5), using a constant \( \epsilon = 0.3 \) (TBC). It is not clear whether it is essential to include the derivatives of \( BC \) in (4). My feeling is that this cannot be critical, because the use of the \( V \) band is quite arbitrary (e.g., the \( G \) band is obviously more ‘bolometric’ than \( V \)).