Basic Angle Stability Specification

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ABSTRACT. A new specification of basic angle stability is proposed, in which a systematic variation of the basic angle with respect to the solar direction is considered separately from random variations.

1 Introduction

The present scientific specification for the stability of the basic angle is 7 $\mu$as rms over 6 hours. In view of the dominant contribution of this basic angle variation coming from thermal fluctuations at the satellite spin period, we should re-examine whether this is correctly specified. In practice, the satellite industrial teams interpret this as being 7 $\mu$as rms specifically at the spin period, in which case the effects would no longer be added in quadrature to the individual scan-level along-scan (‘abscissa’) positional error, but more strongly correlated over a number of successive scans.

Basic angle variations are not explicitly included in the accuracy model so far, but are included in an overall calibration error. The accuracy model follows exactly Sections 7.4.3 and 7.4.4 of the CTSR (p. 261). For Gaia-2, the overall calibration error $\sigma_{\text{cal}}$, added in quadrature to the single-CCD centroiding error $\sigma_{\xi}$, is 40 $\mu$as.

This document presents a simple analysis of the propagation of basic-angle variations into stellar parallaxes, leading to a revised specification in which certain components of the ‘systematic’ (correlated) variations are considered separately.

2 Error propagation model

2.1 Propagation from basic angle to abscissa

General basic-angle variations can be separated into low-frequency components (i.e., on time scales longer than the spin period), which are calibrated as part of the normal data analysis (GIS), and high-frequency components, which (in general) cannot be calibrated. A reasonable model for the high-frequency variations is to consider a harmonic series in terms of the spin phase $\Omega$ (where we put $\Omega = 0$ when the $x$ axis [half-way between the two Astro fields] is closest to the Sun):

$$\Delta \gamma = \sum_{k=1}^{\infty} a_k \cos(k\Omega) + b_k \sin(k\Omega)$$

(1)
where $\gamma(t) = \gamma_0 + \Delta \gamma$ is the basic angle. Note that the coefficients $a_k$, $b_k$ need not be the same on successive spins (although they would have to satisfy a continuity condition). Low-frequency variations (implicit in $\gamma_0$) are not further considered.

A basic-angle variation of the form (1) induces a systematic error in the stellar abscissae $v$ along the scanning great circle which can also be described by a harmonic series:

$$\Delta v = \sum_{k=1}^{\infty} A_k \cos(kv) + B_k \sin(kv) \quad (2)$$

where $v = 0$ is the abscissa of the Sun. Since $v_p \simeq \Omega + \gamma/2$ and $v_f \simeq \Omega - \gamma/2$ in the preceding and following fields, and using that $\Delta \gamma = \Delta v_p - \Delta v_f$, we obtain the following relations between the harmonic coefficients:

$$A_k = -\frac{b_k}{2 \sin(k\gamma/2)^2}, \quad B_k = \frac{a_k}{2 \sin(k\gamma/2)} \quad (3)$$

Thus, each harmonic of $\Delta \gamma$ is attenuated (or amplified) by a certain factor depending on the order ($k$) of the harmonic. For the first few harmonic orders, and $\gamma = 99.4$ deg, the factor is:

$$k \quad = \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$$

$$\frac{1}{2 \sin(k\gamma/2)^2} \quad = \quad 0.656 \quad 0.507 \quad 0.974 \quad 1.552 \quad 0.536 \quad 0.567 \quad 2.385 \quad 0.819 \quad (4)$$

Although certain harmonics (e.g., $k = 4$ and 7) are accentuated, the factor is generally less than 1.

The variance of the basic-angle induced abscissa errors is given by

$$\sigma_v^2 = \frac{1}{2} \sum_{k=1}^{\infty} (A_k^2 + B_k^2) = \frac{1}{2} \sum_{k=1}^{\infty} \frac{a_k^2 + b_k^2}{4 \sin^2(k\gamma/2)} \quad (5)$$

If the power spectral density $B(u)$ of the basic-angle fluctuations is known (with $u$ denoting the temporal frequency) and assumed to be smooth, one can estimate the expected value of $\frac{1}{2}(a_k^2 + b_k^2)$ as $B(k/T)/T$, where $T$ is the spin period. Thus, the expected abscissa variance can also be computed as:

$$\sigma_v^2 = \frac{1}{T} \sum_{k=1}^{\infty} \frac{B(k/T)}{4 \sin^2(k\gamma/2)} \quad (6)$$

2.2 Propagation from abscissa to parallax

2.2.1 Random part

According to Gaia–JdB–012 the average ‘coefficient of improvement’ from FOV transit to parallax is

$$\langle \text{COI}_\pi \rangle = g\pi N_{\text{transit}}^{-1/2} \quad (7)$$
where \( g_\pi = 1.94 \) for \( \xi = 50^\circ \). For the current Gaia-2 design we have \( N_{\text{transit}} = 90 \) and consequently \( \langle \text{COI}_\pi \rangle = 0.204 \).

However, this COI does not immediately apply to the RMS abscissa error \( \sigma_v \) in (5). The reason is that successive FOV transits separated by less than a spin period (6 hr) should only count as one ‘abscissa’ measurement. Simulations show that the mean number of field transits per such absissa is 1.45. Therefore, the relevant coefficient to be applied to the RMS abscissa error is \( \langle \text{COI}_\pi \rangle \sqrt{1.45} = 0.246 \). Thus, for the random part of the basic-angle variations we have

\[
\sigma_\pi = 0.246 \sigma_v
\]

where \( \sigma_v \) is given by (5) or (6).

2.2.2 Systematic part

It is well known that a parallax shift of \( \Delta \pi \) corresponds to a shift in the abscissa by

\[
\Delta v = \Delta \pi \sin \xi \sin v
\]

where \( \xi \) is the Sun–spin axis angle. Thus, a global abscissa error of the form \( B_1 \sin v \) is indistinguishable from a systematic shift (zero-point error) of all parallaxes by

\[
\Delta \pi = \frac{B_1}{\sin \xi}
\]

From (3) we see that such an effect would be produced by a global basic-angle error of the form \( a_1 \cos \Omega \), viz.:

\[
\Delta \pi = \frac{a_1}{2 \sin \xi \sin(\gamma/2)} = 0.856 a_1
\]

with current parameters \( (\xi = 50 \text{ deg}, \gamma = 99.4 \text{ deg}) \). A basic-angle variation of this form would thus be impossible to calibrate internally in GIS, i.e., without resorting to astrophysical assumptions or relying on the basic-angle monitoring device (which of course might be susceptible to a similar effect).

No other kind of (solar-direction related) abscissa error has a perfect correlation with any of the astrometric parameters. Thus all other variations can be treated as random errors, although their effects may be correlated on the sky.

An rms basic-angle variation of 7 \( \mu \text{as} \) having the particular form considered above would produce a global parallax shift of 8.6 \( \mu \text{as} \). This is of course totally unacceptable, illustrating the need for a more stringent specification.

3 Derived specifications

A significant global shift of all parallaxes would be extremely detrimental to the scientific goals of Gaia, since many of them depend on Gaia’s capability to deliver ‘absolute’ parallaxes. In fact the requirement of ‘absoluteness’ must be set an order of magnitude below
the random errors for bright stars. Given that the ‘accuracy floor’ for bright stars will be around 4 \mu as, we require $|\Delta \pi| \leq 0.4$ or, using (11),

$$|a_1| \leq 0.5 \mu as$$

(12)

Even if $a_1$ does not remain constant over the mission, it is conceivable that correlated variations of this kind could produce a regional parallax bias. Thus, (12) should apply to the basic-angle variation over any time interval longer than (say) a day. Allowing a statistical distribution of $a_1$ means that the upper limit in (12) may be interpreted as a one-sigma specification (per day).

For all other basic-angle variations the random propagation model in (5)–(8) is applicable. The corresponding requirement is

$$\sigma_\pi \leq 1.2 \mu as$$

(13)

Note that this corresponds to an RMS basic angle fluctuation of $(1.2 \mu as)/(0.246 \times 0.656) = 7.4 \mu as$ if all the power is in the spin frequency ($k = 1$).

4 Conclusion

The proposed new specification of the basic-angle stability consists of two parts:

1. general basic-angle variations (considered as a random process) should satisfy the requirement (13), with $\sigma_\pi$ computed from (5)–(8);

2. in addition, the coefficient $a_1$ of the $\cos \Omega$ term fitted to the basic-angle variations over any 1 day period should satisfy $|a_1| \leq 0.5 \mu as$ ($1\sigma$).

Item 1 is nearly equivalent to the previous specification of 7 \mu as RMS over 6 hours. Item 2 implies a tightening of the specification for one particular component of the basic-angle variations, which however is well motivated with our improved understanding of the potentially harmful effects of such variations.