GAIA chromaticity calibration
and design of the Broad Band Photometry system

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ABSTRACT. The chromaticity calibration for GAIA astrometry and the choice of filters for the Broad Band Photometer (BBP) are discussed. One aim is to define a procedure for the chromaticity calibration, another to derive the corresponding requirements on the BBP filter system. Main results: (1) an improved algorithm to compute the monochromatic line spread function was found; (2) the choice of centroiding method has a major impact on the chromaticity error and a compromise between precision and chromaticity may be necessary; (3) the chromaticity should be calibrated in terms of linear or quadratic functions of the integrated fluxes through the filters, not in terms of colour indices; (4) the accuracy could be enhanced significantly by the use of external aberration information (from WFE sensors) and spectral synthesis methods; (5) non-standard passband shapes (overlapping triangular or bell-shaped bands) are preferred for the chromaticity calibration; (6) four BBP bands may be sufficient for this purpose. The performance of idealised systems and procedures are studied by ‘calibrating’ the chromaticity against a set of standard stellar spectra, and then assessing the ‘prediction errors’ for quasar spectra. RMS prediction errors below 40 μas (per elementary observation) are reached with very pessimistic and unknown wavefront errors. With perfectly known WFE the RMS prediction error could be reduced to 5 μas for all kinds of spectra.

1 Introduction

In the presence of asymmetric aberrations, the centroid of the diffraction image of GAIA is wavelength-dependent. This is simply a consequence of that the diffraction effect itself depends on the wavelength. The polychromatic image centroid is therefore shifted by an amount which depends both on the aberrations (which vary across the astrometric field) and on the spectral energy distribution (which varies with the source). In addition, the shift could depend on details of the centroiding algorithm. In order to achieve the required astrometric accuracy, the shift must be adequately calibrated as function of all these factors. How to achieve this may be called the chromaticity problem.

Knowledge of the spectral energy distribution (SED) must come primarily from the Broad Band Photometer (BBP), which has the same spatial resolution as the astrometric CCDs and which provides photometric information on all astrometrically observed sources. Some knowledge of the wavefront errors (WFE) across the field could also be available, but in general the chromaticity would have to be determined empirically as function of position in the field as well as versus time.
In previous notes (SAG–LL–16 and SAG–LL–24 by Lindegren; SAG–MV–03 by Vannier) the centroid displacement was examined as function of colour indices defined in the conventional way and using a predefined set of filter bands similar to the Sloan bands. The chromaticity was assessed in terms of the RMS residuals of a polynomial regression of the displacement versus linear combinations of colour indices.

In this note some new ideas are introduced and explored:

- the number and shape of filter passbands are varied in order to optimize the chromaticity calibration, without regard of any astrophysical uses of the BBP (at some later point, astrophysical considerations should of course also be taken into account);

- the chromaticity must be calibrated essentially by means of ordinary stars, but should then be applicable to objects with arbitrary spectra. In other words, the calibration should have good predictive power, which we assess by applying it to quasar spectra;

- it can be argued that, for the present purpose, the BBP should provide a good sampling of the SED, rather than measure specific spectral features;

- expressing the chromaticity as a function of colour indices is physically unsound, and is here replaced by linear or quadratic functions of normalized fluxes;

- a synthetic spectrum approach is used to assess the value of independent WFE data.

The purpose of this note is not to reach a definitive conclusion concerning the choice of BBP filters, nor on the detailed calibration procedure, but to make a first exploration of these ideas.

## 2 Preliminaries

### 2.1 Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$\xi$</td>
<td>along-scan angular coordinate ($\xi_0 =$ centroid coordinate)</td>
</tr>
<tr>
<td>$L(\xi)$</td>
<td>line spread function (LSF) including pixel and TDI smearing</td>
</tr>
<tr>
<td>$s$</td>
<td>standard width of the gaussian fitted to the source profile</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength</td>
</tr>
<tr>
<td>$f(\lambda)$</td>
<td>source spectral energy distribution (SED) in photons per unit $\lambda$</td>
</tr>
<tr>
<td>$T(\lambda)$</td>
<td>optical transmittance of the instrument excluding filters</td>
</tr>
<tr>
<td>$t_i(\lambda)$</td>
<td>optical transmittance of the $i$th BBP filter ($i = 1 \ldots n$)</td>
</tr>
<tr>
<td>$Q(\lambda)$</td>
<td>quantum efficiency of the detector</td>
</tr>
<tr>
<td>$E(\lambda) = f(\lambda)T(\lambda)Q(\lambda)$</td>
<td>‘effective spectrum’ of the source (cf. SAG–LL–025)</td>
</tr>
<tr>
<td>$R_i(\lambda) = t_i(\lambda)T(\lambda)Q(\lambda)$</td>
<td>response function of the $i$th filter band</td>
</tr>
<tr>
<td>$\Phi_i = \int R_i(\lambda)f(\lambda) , d\lambda$</td>
<td>photon flux in the $i$th filter band</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>normalized flux in the $i$th filter band ($\phi_1 + \cdots + \phi_n = 1$)</td>
</tr>
<tr>
<td>$S(\lambda)$</td>
<td>synthesized effective spectrum</td>
</tr>
<tr>
<td>$B_k(\lambda)$</td>
<td>$k$th basis function for the synthesized spectrum ($k = 1 \ldots m$)</td>
</tr>
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</table>
2.2 Calculating the monochromatic LSF

Two previous reports (SAG-LL-24 and 25, October/December 1998) contained detailed recipes for the calculation of the PSF and LSF including aberrations. In the second note the use of the FFT was introduced to speed up the calculation. Those codes are however not quite suitable for very precise calculations, when we want to look at systematic centroid displacements of millipixels or less. There are several reasons for this: first, the calculations were made in single precision; second, the discretization of the pupil introduced wavelength-dependent errors that are not negligible. This is most clearly shown in Fig. 1, where the centroid displacements are plotted as functions of wavelength for a sequence of monochromatic LSF calculated according to SAG-LL-25. The centroiding algorithm uses the gaussian kernel defined in Eq. (9) of SAG-LL-24 with the indicated standard widths $s$ (in pixels).

The saw-tooth ripples in Fig. 1, which for the longest wavelengths have a peak-to-valley amplitude of 0.012 pixel, are caused by the discretization. The linear separation of the points in the pupil plane is proportional to the wavelength, which means that the points fall differently with respect to the pupil edges depending on the wavelength. The discontinuities in the ripples occur at wavelengths where one of the points fall exactly on the edge.

This effect can of course be reduced by increasing the number of discretization points, but that number is already quite large in Fig. 1 ($N_x = 2048$), and the amplitude of the effect only reduces in inverse proportion to $N_x$. A better way would be to introduce a variable weight to each pixel in proportion to the pupil area represented by the point. This was tried, but did not completely eliminate the effect. Instead, a rather different approach was found, to be described hereafter.

It is clear that the cause of the ripples could be eliminated if the discretization of the pupil could be kept fixed as function of wavelength. There would still be a truncation error due to the finite discretization step, but at least that would change only slowly (if at all) with wavelength. The fundamental equation which governs the spacing of the points is Eq. (6) in SAG-LL-25:

$$N_x \Delta x \Delta \xi = N_y \Delta y \Delta \eta = \lambda$$

(1)

where $N_x, N_y$ are the number of discretization points in the along-scan ($x$) and across-scan ($y$) directions; $\Delta x, \Delta y$ are the point spacings in the pupil plane (in the same unit as the wavelength $\lambda$); and $\Delta \xi, \Delta \eta$ are the point spacings in the image plane (in radians). It is seen that the proportionality $\Delta x \propto \lambda$ follows from the requirement to keep $\Delta \xi$ fixed (in this case to 0.25 pixel). If instead we can accept a variable $\Delta \xi$, it would be possible to keep $\Delta \xi$ fixed and avoid the ripples. This means that $\Delta \xi$ must be proportional to $\lambda$.

Having $\Delta \xi \propto \lambda$ would seem to be a severe disadvantage, because it means that the computed LSF must be interpolated to the required points (e.g. at quarter-pixel intervals),
resulting in interpolation errors instead. However, we are here helped by the sampling theorem, which states that the entire information is contained in the sampled signal, provided the sampling frequency is at least twice the bandwidth of the signal. Moreover, the exact continuous signal can then be recovered by trigonometric interpolation (see below). In our case the signal (monochromatic LSF) is indeed bandwidth limited: the maximum spatial frequency is $D_x/\lambda$, if $D_x$ is the pupil size along scan. To retain the full information on the LSF it is thus only necessary that $\Delta \xi \leq \lambda/(2D_x)$. Comparison with Eq. (1) shows that this is equivalent to

$$N_x \Delta x \geq 2D_x .$$  \hspace{1cm} (2)

Since $D_x = 1.7$ m, a good discretization of the pupil and WFE, e.g, with a step less than 0.01 m, should be possible with a moderate number of points, e.g. $N_x = 512$ and $N_y = 256$. The resulting angular step $\Delta \xi = \lambda/(2D_x)$ varies from about 0.5 to 1.8 pixels for $\lambda$ between 300 and 1100 nm.

Provided that the sampling condition (2) is satisfied, the exact values of the LSF can be

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Centroid displacement calculated as function of wavelength for the 'gaussian' location estimator introduced in SAG-LL-24, using four different standard widths ($s = 0.5$ to 2.0 pixels). In this case, the monochromatic line-spread function (LSF) was computed with the FFT algorithm described in SAG-LL-25, using $N_x = 2048$ discretization points in the along-scan direction. The ripples visible especially for the long wavelengths is an effect of the discretization discussed in the text. The wavefront errors assumed here are those of field point $\# 15$, as defined in SAG-LL-25.}
\end{figure}
Figure 2. Same as Fig. 1, but using the improved algorithm in psf.f to compute the monochromatic LSF (fixed pupil discretization followed by trigonometric interpolation in the image plane).

obtained from the sampled values by means of the trigonometric interpolation formula

$$L(\xi) = \sum_{k=-\infty}^{\infty} L(k\Delta\xi) \text{sinc} \left[ \pi \left( k - \frac{\xi}{\Delta\xi} \right) \right].$$  \hspace{1cm} (3)

In practice the summation must be truncated to a finite range of $k$. It may be tempting to limit the summation to a range of $k$ values for which the sinc function is 'significant'. However, it should be noted that $L(\xi)$ falls off much more quickly at large angles from its centre than does the sinc function. Thus it is important to include in the summation a sufficient range to cover the central parts both of the sinc function and the LSF.

The program psf.f computes the monochromatic LSF for a range of wavelengths by this method. For given WFE, it computes the two-dimensional monochromatic optical PSF by means of the FFT, then obtains the optical LSF by summing over $\eta$, convolves with the pixel size and TDI smearing using the one-dimensional FFT, and finally interpolates to a fixed angular sampling frequency by means of Eq. (3). Figure 2 shows that this procedure successfully removed the ripples in Fig. 1, at the same time as the computations were speeded up by a factor seven in this example.
2.3 Rôle of the location estimator

There are several interesting aspects of Fig. 2 which should be commented on. First, it can be noted that the variation of the monochromatic location with wavelength is a very smooth and almost monotonic function. The polychromatic location is not simply a flux weighted mean of the monochromatic locations, but depends on the polychromatic LSF in a more complex and non-linear fashion. Nevertheless, the simplicity of these curves makes it likely that the chromaticity does not depend on fine structures of the spectrum, but rather on its global characteristics. This gives hope that the BBP, with suitably chosen filters, should allow a very exact calibration of the effect.

Second, there is a considerable difference in location between the different estimators, especially at the short wavelengths. This is problematic, because the optimum location estimator, from the viewpoint of precision (random noise), depends e.g. on the magnitude through the variation in signal-to-noise ratio, and on the colour index through the width of the LSF. If the location estimator is made to depend on these factors, as in the routine \texttt{1ocest} (SAG-LL-32), then the chromaticity may become a complex function not only of the spectrum, but also of the magnitude, background level, readout noise, and so on. Quite possibly, this could prevent the practical calibration of the effect to sufficient accuracy. Alternatively, the location estimator could be kept fixed for all sources, independent of magnitude etc., but then it will usually be slightly suboptimal and other bias effects could appear instead. This is a point to consider for further optimisation.

Third, it can be noted that the variation with $\lambda$ is much smaller for $s = 2$ than for the smaller $s$. This is not only the case for the particular WFE in Fig. 2, but is a general feature. Recall that $s$ is the standard width (in pixels) of the gaussian fitted to the LSF. Actually the limiting values $s = 0$ and $s = \infty$ correspond to the locations of the mode (maximum) and mean (centre of gravity) of the LSF. It is not surprising that the mean is less dependent on wavelength than the mode (cf. Fig. 1 in SAG-LL-24). However, it can be shown that \textit{the mean is in principle completely achromatic}, i.e. in the limit of large $s$ the curve would be completely flat. (The centre of gravity corresponds to the mean slope of the wavefront over the pupil. This is also the limiting value obtained with \textit{any} symmetric estimator for $\lambda \to \infty$.) Thus, the chromaticity could in principle be eliminated by using the centre of gravity as the location estimator. This is not possible in practice because it would be too sensitive to background errors and similar. However, as indicated by the example in Fig. 2, it may be possible to reduce the chromaticity problem considerably by a suitable choice of the location estimator. Again, this would not be optimal for precision, so a compromise would have to be found.

In the following all calculations are made using the gaussian location estimator with $s = 1.5$, which for typical LSF and signal-to-noise ratios is not too far from optimal with respect to precision. The wavefront errors at field point #15 (as defined in SAG-LL-25) are used, although these are probably \textit{very} pessimistic: the RMS WFE is 85 nm for this field point, which gives a Strehl ratio of 0.56 at 700 nm wavelength. A more typical figure is 35 nm RMS (Strehl ratio 0.91), which could reduce the chromaticity by an order of magnitude compared with the present results. However, it was felt important to develop procedures that work even in the (non-linear) region of moderately strong aberrations.
2.4 Sample spectra

The spectral energy distributions considered here are the 175 stellar spectra from Gunn & Stryker (ApJS 52, 121, 1983) and 101 quasar spectra corresponding to a single mean spectrum redshifted by $z = 0(0.05)5$. The mean quasar spectrum was constructed by combining the mean spectrum of Zheng et al. (ApJ 475, 469, 1997) for $\lambda_{\text{rest}} < 250$ nm with that of Brotherton et al. (ApJ 546, 775, 2001) for $\lambda_{\text{rest}} > 100$ nm. In the overlapping region (100 to 250 nm) a gliding mean was used. For $\lambda_{\text{rest}} > 830$ nm a featureless continuum with slope 0.635 (in $\nu L_\nu$ versus $\nu$) was assumed; cf. Elvis et al. (ApJS 95, 1, 1994). The resulting mean quasar spectrum is shown in Fig. 3.

A synthetic colour index $V - I$ was also computed for each spectrum, using the passbands from Bessell (PASP 102, 1181, 1990). In the top panel of Fig. 4, the centroid displacement is plotted versus $V - I$ for the whole set of stellar and quasar spectra. The total range of displacements is about 0.2 pixel (7500 $\mu$as) for the stellar spectra, and 0.04 pixel (1500 $\mu$as) for the quasar spectrum. The lower panels show the residuals of a polynomial of degree 4 fitted to the stellar data. The RMS residual is 132 $\mu$as for the stellar data and 813 $\mu$as for the quasar data. The figure shows clearly that this particular colour index has little relevance for the chromatic displacement of quasars, although it would probably allow to establish a reasonable mean calibration for ordinary stars.

In the bottom panel of Fig. 4 and in subsequent plots, ‘Spectrum number’ runs from 1 to 175 for the Gunn & Stryker stellar spectra, and from 0 to –100 for the mean quasar spectrum with $z = 0$ to 5.
Figure 4. Top: The centroid displacement (using $s = 1.5$ pixel) versus colour index $V-I$ for a range of stellar and quasar spectra. Middle and bottom: Residuals from a fourth-degree polynomial in $V-I$ fitted to the stellar data. Filled circles: residuals for the stars. Open circles: prediction errors for the QSOs.
3 Methods of chromaticity calibration

3.1 Colour indices or what?

In previous studies the displacement of the polychromatic centroid was computed for a range of spectral energy distributions and compared with various colour indices from the broad band filters. The purpose was to find an index, or a combination of indices, for which there was a simple one-to-one relation to the centroid displacement. The chromaticity would then be calibrated e.g. in terms of a polynomial in the combined index. Ideally, this index should work for arbitrary spectral shapes and not just for ordinary stellar spectra. It is particularly important that emission line spectra such as from quasars are adequately treated.

A simple thought experiment shows that the ordinary astronomical colour indices (basically logarithmic flux ratios) are in fact not at all suitable for chromaticity calibration. Consider an arbitrary colour index $c = a - b$, where $a$ and $b$ are the magnitudes in two disjoint wavelength bands. Now suppose we have a peculiar spectrum in which the flux is effectively blocked (by some unknown astrophysical process) in the $a$ band; then $c$ tends to infinity while the centroid displacement tends to whatever (well-defined) value it has in the $b$ band. Thus, if the displacement is calibrated by means of ordinary stellar spectra in terms of a polynomial in $c$, then this calibration will inevitably fail for this hypothetical spectrum. The same reasoning applies to any linear combination of such colour indices.

As mentioned previously, one cannot assume that the polychromatic centroid displacement is just a linear combination of the monochromatic displacements weighted by the spectrum. However, it is intuitively clear that such an assumption, although not strictly correct, is more likely to work, for arbitrary spectra, than the logarithmic colour indices. It would, for instance, give the correct behaviour in the extreme case of a spectrum consisting of a single emission line. This suggests that the chromaticity should be calibrated in terms of the observed fluxes in the photometric bands, rather than the magnitudes.

The flux in the $i$th photometric band is

$$
\Phi_i = \int t_i(\lambda)T(\lambda)Q(\lambda)f(\lambda)\,d\lambda, \quad i = 1 \ldots n, \quad (4)
$$

where $n$ is the number of BBP bands (Only $n = 4$ or $5$ is considered below.) The normalized fluxes are

$$
\phi_i = \frac{\Phi_i}{\Phi_1 + \Phi_2 + \cdots + \Phi_n}, \quad i = 1 \ldots n. \quad (5)
$$

Like ordinary colour indices the normalized fluxes do not depend on the brightness of the source, but only on the spectral distribution. Note that the normalized fluxes are strictly bounded, $0 \leq \phi_i \leq 1$, and that $\phi_1 + \cdots + \phi_n = 1$. 


3.2 Linear regression

As a first approximation, a linear regression formula is tried for the centroid position,

\[ \xi_0 \sim \sum_{i=1}^{n} x_i \phi_i \]  

(6)

(the sign ‘\( \sim \)’ means ‘least-squares approximated by’). Here \( x_i \) are \( n \) coefficients to be determined in the calibration process.\(^1\) In the core processing, the coefficients would be introduced as unknowns. Additional formalism is necessary to take into account temporal and spatial variations of the chromaticity, but that aspect of the calibration is not considered here.

3.3 Quadratic regression

If the linear model is not good enough, second-order terms could be added. The simplest quadratic version is

\[ \xi_0 \sim \sum_{i=1}^{n} x_i \phi_i + \sum_{i=1}^{n} x_{n+i} \phi_i^2 \]  

(7)

with \( 2n \) coefficients. As shown below, this model seems to work surprisingly well, so we will not introduce more complex versions with cross-terms.

Subsequently, Eq. (6) or (7) will be used to fit the centroid displacement for the whole range of Gunn & Stryker stellar spectra. The resulting coefficients are then used to predict the displacement for the mean quasar spectrum at various redshifts. The RMS prediction error is taken as a figure of merit for the filter combination.

3.4 Spectral synthesis and the use of independent WFE data

The linear and quadratic regression methods are purely empirical and do not require any data at all on the wavefront errors. Clearly, knowledge about the WFE does however represent additional information which could be used to improve the chromaticity calibration. In this section we consider this problem in a somewhat idealized setting.

The astrometric fields of GAIA are equipped with wavefront sensors whose main purpose is to align the mirrors at the beginning of the mission. In principle, this means that the WFE could be measured at arbitrary times at specific points in the field, and perhaps interpolated to other points by means of a computer model of the optics. This would of course have its own calibration problems, which have to be ignored for the moment. It is however interesting to pose the question: Assuming that the WFE is perfectly known,\(^\text{1}\)

\(^\text{1}\)Since only \( n - 1 \) of the numbers \( \phi_i \) are linearly independent, it would seem natural to use only \( n - 1 \) of them in the regression formula. However, a constant term would also be required, thus: \( \xi_0 = y_0 + y_1 \phi_1 + \cdots + y_{n-1} \phi_{n-1} \). But this is equivalent to (6) with \( x_i = y_0 + y_i, i = 1 \ldots n - 1 \) and \( x_n = y_0 \). The form (6) is preferred for its symmetry.
Figure 5. The solid curves show the effective spectrum (photon flux times quantum efficiency) for stellar spectrum # 132 (spectral type K3III). The dashed curves are the synthetic spectra obtained by matching the fluxes in two different filter systems: BBP3G (top) and S4LOG (bottom).

how well could we compute the centroid displacement for an arbitrary source? In this case the measurements from the BBP must be used to estimate the SED, from which the centroid can be predicted as in Section 2. The RMS prediction error (for stars as well as quasars) can be used as a figure of merit for the filter system.

In this case we need to parameterize the SED in some fashion. For practical reasons it is probably necessary to use the same (continuous) SED model for all kinds of sources. A spline model, similar to what was used in SAG–LL–25, might be a suitable choice. We have used a model spectrum described by a continuous polygon with 10 degrees of freedom:

\[ E(\lambda) \sim S(\lambda) \equiv \sum_{k=0}^{m} c_k B_k(\lambda) \quad (m = 10). \]  

(8)
Figure 6. Same as Fig. 5 but for the mean quasar spectrum at redshift $z = 2.5$.

The linear basic splines $B_i(\lambda)$ have the knot sequence 300, 345, 396, 455, 524, 602, 692, 795, 914, 1050 nm (forming a geometric series) with free ends. That is, the values at these 10 wavelengths are free parameters (equal to $c_k$).

The 10 parameters $c_k$ are chosen such that the synthetic effective spectrum $S(\lambda)$ gives the same fluxes $\Phi_i$ through the $n$ filters as the real effective spectrum $E(\lambda)$. This results in the $n$ linear equations

$$\sum_{k=1}^{m} A_{ik} c_k = \Phi_i ,$$  \hspace{1cm} (9)

where

$$A_{ik} = \int t_i(\lambda) B_k(\lambda) d\lambda .$$  \hspace{1cm} (10)
Equation (9) is clearly underdetermined (since \( n < m \)), but a minimum-norm solution can be found by using the pseudo-inverse of the matrix \( A \):

\[
c = A^+ \Phi.
\]

(11)

A double-precision version of the SVD algorithm \texttt{svdcmp} from Numerical Recipes is used to compute \( A^+ \) once for a given filter system.

The synthesized spectrum \( S(\lambda) \) is the ‘simplest’ one, of the form (8) and consistent with the observed fluxes, in the sense that \(|c|\) is minimal. Figures 5 and 6 are examples of the true and synthesized effective spectra for two different filter systems. The question is which system generally provides synthesized spectra that give the most accurate centroid prediction. This is assessed from the RMS prediction error for the stellar and quasar spectra.

4 Filter systems considered

Several passband combinations using four or five broad-band filters are considered. The BBP part of the 3G system, using five filters (as defined in SAG–CUO–78), is also considered as a reference and called BBP3G hereafter.

Other filter sets are designated in the following way: Xn\{scale\}, where X denotes the filter type (see below), \( n = 4 \) or \( 5 \) is the number of filters used, and \{scale\} defines the method to divide the wavelength scale (LIN or LOG).

\{scale\} = LIN means that the central wavelengths are equidistant on the linear wavelength scale. Thus the sampling interval as well as the filter bandwidths are independent of \( \lambda \).

\{scale\} = LOG means that the filters are equidistant in log \( \lambda \), and the bandwidths also constant in log \( \lambda \). Thus both the sampling interval and bandwidths are proportional to \( \lambda \). Due to the shape of the curves in Fig. 2 the logarithmic scale may be better than the linear scale.

As for the filter type X, the following variants are considered (Fig. 7):

R: rectangular filter bands covering 300 to 1050 nm without gaps or overlap.

C: cosine-squared filter bands with spacing equal to the FWHM (thus overlapping). This can be seen as a ‘smoothed’ version of R.

T: triangular filter bands with spacing equal to the FWHM (thus overlapping). The two extreme bands have their full support within the the region 300 to 1050 nm.

U: triangular filter bands with spacing equal to the FWHM (thus overlapping), but the two extreme bands have their maximum transmittance at 300 and 1050 nm.

S: a ‘Shannon-type’ system — cosine-squared filter bands satisfying the spectral sampling condition, i.e. sampling interval equal to half the FWHM, also in the extreme bands. See Young (A&A 288, 683, 1994) for a discussion of such filter systems.
Figure 7. Schematic representation of the five filter system types considered in this note: R, C, T, U and S. Filter transmittance curves starting at the short wavelength cut-off are shown for a constant sampling interval in wavelength. In reality, the sampling interval varies, for a fixed number of bands, depending on the type. More precise response functions (including the CCD quantum efficiency) are shown in Fig. 8.

5 Results

In total 21 different filter systems were considered, including BBP3G. The response functions $R_i(\lambda)$ are shown in Fig. 8. The assumed quantum efficiency curve of the CCD is that of CCD #1B, as was used for the astrometric fields in the GAIA study report (Table 3.13).

In subsequent figures (9 through 29) the performance of each filter system is shown in the form of three plots containing the residuals and prediction errors for the three calibration methods described above: linear regression (top diagram), quadratic regression (middle) and spectral synthesis (bottom). As in the bottom diagram of Fig. 4, the errors are plotted versus the spectrum number defined in Sect. 2.4 (positive for stars, non-positive for quasars).

The RMS residuals and prediction errors are summarized in Table 1.
Table 1. RMS residuals (res) and prediction errors (pred) in μas.

<table>
<thead>
<tr>
<th>Filter system</th>
<th>linear stars (res)</th>
<th>quadratic stars (res)</th>
<th>synthesis stars (pred)</th>
<th>linear QSOs (res)</th>
<th>quadratic QSOs (pred)</th>
<th>synthesis QSOs (pred)</th>
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</thead>
<tbody>
<tr>
<td>BBP3G</td>
<td>34.8</td>
<td>127.9</td>
<td>23.6</td>
<td>108.8</td>
<td>205.3</td>
<td>97.8</td>
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<td>R4LIN</td>
<td>56.3</td>
<td>114.0</td>
<td>24.0</td>
<td>115.4</td>
<td>63.4</td>
<td>95.4</td>
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<td>R4LOG</td>
<td>46.9</td>
<td>157.7</td>
<td>29.0</td>
<td>93.8</td>
<td>128.4</td>
<td>133.8</td>
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<td>14.4</td>
<td>75.2</td>
<td>32.8</td>
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<td>14.1</td>
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<td>C5LIN</td>
<td>27.9</td>
<td>128.0</td>
<td>12.3</td>
<td>49.3</td>
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6 Conclusions

The quadratic regression is clearly much better than the linear. That it improves the fit for the stellar data is not surprising, as there are twice as many free parameters; but the prediction errors for the quasar spectra are also very much reduced, which shows that the quadratic model is actually better. We should therefore disregard the results from the linear regression model when assessing the different filter systems.

Figure 30 shows that there is a good correlation between the results obtained with the quadratic model on one hand, and the spectral synthesis method on the other. In both cases the S and U type systems are superior to the other systems. A common feature of the S and U systems is that they sample the extreme wavelengths better than the other systems do. Based on Fig. 2 one can assume that it is particularly important to sample the short-wavelength end of the spectrum. This is also indicated by the fact that the LOG scale is – usually – somewhat better than the LIN scale.

A somewhat surprising result is that using five bands does not give a clear advantage over four bands. At least the S4 systems are almost as good as the best S5 or U5 systems.
Figure 30. Graphical summary of the predictive performance of the 21 filter systems. The systems in the lower-left corner are best from the viewpoint of chromaticity calibration.

Again, this could be a consequence of the relative simplicity of the curves in Fig. 2: perhaps it does not take more than four points to describe the wavelength dependence adequately.

The experiments support the conjecture in GAIA–LL–36, namely that an adequate sampling of the spectrum, with overlapping bell-shaped (or triangular) passbands, is important for the chromaticity calibration. Tentatively, filter systems such as S5LOG, U5LOG, S4LOG, S4LIN or U5LIN could be the basis for further optimization and consideration also from the astrophysical viewpoint.

The ability to determine reliable photometric redshifts for quasars could be another criterion for the BBP filter system. Although that aspect has not been considered here, it could be expected that systems with gradually overlapping bands are better for this purpose than, e.g., rectangular bands. Quite possibly the chromaticity and the redshift determination could lead to similar designs, although five bands should be better for the redshifts than four.

The vast improvement in prediction errors obtained with the spectral synthesis method is of course based on the unrealistic assumption that the WFE are accurately known. However it indicates that the use of independent WFE information needs to be considered. Some combination of the spectral synthesis method with the empirical regression method should be found, which poses some interesting problems for further study.

(End of text: 28 pages of diagrams follow)
FIGURE 8A. Response functions for the considered BBP filter systems.
Figure 8b. Response functions for the considered BBP filter systems (continued).
Figure 8c. Response functions for the considered BBP filter systems (continued).
FIGURE 8D. Response functions for the considered BBP filter systems (continued).
Figure 8e. Response functions for the considered BBP filter systems (continued).
Figure 8f. Response functions for the considered BBP filter systems (continued).
Figure 8G. Response functions for the considered BBP filter systems (continued).
Figure 9. Residuals and prediction errors for filter system BBP3G. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 10. Residuals and prediction errors for filter system R4LIN. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 11. Residuals and prediction errors for filter system R4LOG. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 12. Residuals and prediction errors for filter system R5LIN. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 13. Residuals and prediction errors for filter system R5LOG. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 14. Residuals and prediction errors for filter system C4LIN. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 15. Residuals and prediction errors for filter system C4LOG. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 16. Residuals and prediction errors for filter system C5LIN. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 17. Residuals and prediction errors for filter system C5LOG. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 18. Residuals and prediction errors for filter system T4LIN. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
FIGURE 19. Residuals and prediction errors for filter system T4LOG. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 20. Residuals and prediction errors for filter system T5LIN. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 21. Residuals and prediction errors for filter system T5LOG. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 22. Residuals and prediction errors for filter system U4LIN. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 23. Residuals and prediction errors for filter system U4LOG. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 24. Residuals and prediction errors for filter system U5LIN. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 25. Residuals and prediction errors for filter system USLOG. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 26. Residuals and prediction errors for filter system S4LIN. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 27. Residuals and prediction errors for filter system S4LOG. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 28. Residuals and prediction errors for filter system S5LIN. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.
Figure 29. Residuals and prediction errors for filter system S5LOG. Top – linear regression; middle – quadratic regression; bottom – spectral synthesis using known WFE.