Choice of basic angles for three viewing directions

SAG.II.010 (L. Lindegren, 1997 October 14)

A previous note (SAG.II.009) gave arguments for the use of three interconnected viewing directions in GAIA, rather than just two. The three viewing directions are connected by three basic angles $\gamma_1$, $\gamma_2$ and $\gamma_3 = \gamma_1 + \gamma_2$. The choice of these angles was not discussed, but the following formula was given for the relative variance of the abscissa harmonics of order $n = 1, 2, \ldots$:

$$u_3(n, \gamma_1, \gamma_2) = \left[ 3 - \sin^2 \left( \frac{nw}{2} \right) \left( 1 + \frac{2}{3} \cos n\gamma_1 + \cos n\gamma_2 + \cos n\gamma_3 \right) \right]^{-1}$$

(1)

Here, $w$ is the effective width of the field of view, assumed to be $w = 2^\circ$. By proper choice of $\gamma_1$ and $\gamma_2$, any given couple of harmonics can be suppressed. However, one cannot completely suppress all harmonics at the same time. To find an optimum pair $(\gamma_1, \gamma_2)$ one needs to define a merit function depending on all the different harmonics. There is no a priori obviously "right" way to calculate the merit function, but fortunately this is not critical for the optimisation.

Several possible merit functions have been tried, parametrized by the following general form:

$$Q(\gamma_1, \gamma_2) = \left[ \sum_{n=1}^N u_3(n, \gamma_1, \gamma_2)^a \frac{\exp(-bn)}{nc} \right]^{1/a}$$

(2)

Here $N = 100$ is the maximum order considered, $a \geq 1$ is a parameter determining the relative importance of strong and weak harmonics, while $b \geq 0$ and $c \geq 0$ determine the relative weight of the different orders according to an exponential or inverse power law. Only a few different combinations of the parameters $a, b, c$ could be tried.

For practical reasons it may be desirable to keep $\gamma_1$ and $\gamma_2$ close to $60^\circ$ and $\gamma_3$ close to $120^\circ$, say, within $\pm A$ in each case. This defines a hexagonal region in the $(\gamma_1, \gamma_2)$ plane to be explored by mapping the function $Q$. Some representative results for $A = 10^\circ$ are shown in Figures 1 to 4. In each case the natural logarithm of $Q$ is shown, with contours at intervals of 0.1 ($\sim 10\%$ in variance or $5\%$ in amplitude).

The four maps are sufficiently similar to allow some rather general conclusions concerning the optimum choice of $(\gamma_1, \gamma_2)$ within the given restrictions. First, $\gamma_1 = \gamma_2 = 60^\circ$ must be avoided — hardly a surprising result. The width of the area to avoid is of the order of $\pm 2^\circ$, probably related to the (assumed) effective field width $w$. Secondly, in the interval $50^\circ \leq \gamma_1 \leq 54^\circ$ we are in a relatively flat region for practically any $\gamma_2 > 60^\circ$, except that there is a weak ridge along the diagonal $\gamma_1 + \gamma_2 \approx 120^\circ$.

An additional consideration may be that it is sensible to avoid values of $\gamma_1$ and $\gamma_2$ which, by themselves (i.e. in a configuration with only two viewing directions), are "bad", e.g. $60^\circ$.

Taking the various considerations into account, it appears that $(54^\circ, 68^\circ)$ is a particularly "good" pair, if $A = 8^\circ$ is an acceptable constraint on the deviation from $60^\circ$. An alternative pair could be $(52^\circ, 66^\circ)$, also consistent with $A = 8^\circ$. It becomes more difficult to find a good pair for smaller $A$, although $(53^\circ, 65^\circ)$, for $A = 7^\circ$, is not unreasonable.
Figure 1. Contour plot of \( \ln Q(\gamma_1, \gamma_2) \) for \( a = 1, b = 0, c = 0 \) (i.e. an unweighted sum of the harmonic variances). The ‘acceptable’ region, for \( A = 10^\circ \), is delimited by the square frame and the two slanted lines.

Figure 2. Contour plot of \( \ln Q(\gamma_1, \gamma_2) \) for \( a = 2, b = 0, c = 0 \) (i.e. an unweighted quadratic sum of the variances, stressing the strong harmonics)
Figure 3. Contour plot of $\ln Q(\gamma_1, \gamma_2)$ for $a = 1$, $b = 0$, $c = 1$ (i.e. a linear $1/n$ weighting of the harmonics).

Figure 4. Contour plot of $\ln Q(\gamma_1, \gamma_2)$ for $a = 1$, $b = 0.2$, $c = 0$ (i.e. an exponential weighting of the harmonics).