Statistical phase error due to parasitic stars

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It can be shown (cf. Padua Colloquium, p. 118) that the rms positional dispersion (or 'dispersion radius'), caused by a parasitic star $\Delta m$ magnitudes fainter than the programme star, is approximately given by $\sigma(\Delta m) = 0.08s\text{dex}(-0.4\Delta m)$, where $s$ is the slit period. This holds for any separation in the range from $\approx 0.5s$ to the IFOV radius. For a programme star of magnitude $M$ it is interesting to calculate the total expected dispersion from all stars fainter than $M$. Let $A$ be the IFOV area (in deg$^2$) and $N(m)$ the density of stars brighter than $m$ (in deg$^{-2}$). The expected number of parasitic stars (not counting physical components!) in the magnitude range $[m, m + dm]$ is $2AN'(m)dm$, where the factor 2 comes from the two FOVs. Their contribution to the error variance is $2AN'(m)dm \times (0.08s)^2\text{dex}[-0.8(m - M)]$. Therefore, the total error variance is

$$\sigma^2 = 2A(0.08s)^2 \int_M^{\infty} N'(m) \text{dex}[-0.8(m - M)] dm$$

Adopting a power law for the mean stellar density, $N(m) = \text{dex}(\alpha + \beta m)$, gives:

$$\sigma^2 = \frac{\beta}{0.8 - \beta}(0.08s)^22AN(M)$$

Example: According to Allen (Astrophysical Quantities 1973, p. 244), $\alpha = -3.1$, $\beta = 0.41$ for visual $m$ between 10 and 15. For Hipparcos-1, $A = 9.10^{-8}$ deg$^2$ and $s = 1208$ mas, which gives:

$$\sigma = (2.6 \text{ mas}) \times \text{dex}[0.205(M - 9)]$$

or 2.6, 11 and 44 mas at $M = 9, 12$ and 15, respectively. One can probably assume that the phase error is statistically independent for each FOV crossing, which means that $\sigma$ must be added quadratically to the photon noise at the FOV crossing level.