Phase errors caused by retaining the first sample after repositioning

by L Lindegren

1. Introduction

It has been found that the intensity of the first IDT sample after repositioning the IFOV to a new star is systematically reduced by almost 10%. Therefore, it has been decided to discard that sample in the IDT data analysis. However, it is not obvious that this is optimal from a global accuracy point of view. To my knowledge no study has been made of the possible adverse effects of retaining the first sample, which after all represents about 2.5% of the total observing time, or almost a full month of the whole mission. Are we really prepared to throw away that much data just on the feeling that it may be useless?

Simple reasoning tells us that the 10% intensity reduction should be negligible as long as the photon noise on a single sample is greater than 10%. This is the case if the expected number of counts per sample is less than 100 photons, corresponding to a star of magnitude 4 or 5. For stars fainter than that we should in fact gain by keeping the sample. This conclusion is supported by a more detailed analytical model, which is described below, and by a direct comparison of the phases for the 19 bright transits in NDAC/IO/134 computed with and without the first sample.

The validity of these arguments relies completely on the assumption (as yet unproven) that the first sample does not introduce a bias on the great-circle level. This should be tested by making two great-circle reductions, one with and one without the first sample. (It has been agreed to make such a test within NDAC.)

A more correct way to deal with the first sample is of course to include the (known) intensity factor in the model fitted to the IDT counts. However, this is probably too complicated to be implemented at this stage. Therefore I consider here only a simple switch, e.g. depending on the magnitude: either include the sample and assume it is OK, or discard it.

2. Analytical model

The analytical model of the phase error caused by the reduced intensity of the first sample is based on an unweighted least-squares fit of the 5-parameter model to the counts, assuming uniform distribution of reference phases (so that the normal equations matrix becomes diagonal). Let \( I_k = B + A(1 + M1 \cos f_k + M2 \cos 2f_k) \) be the intensity for sample \( k = 2, 3, \ldots, n \) in an interlacing period, while the intensity of the first sample is \( (1-x)I_k \), where \( x \) is the intensity reduction. With \( R = M2/M1 \) and \( Q = 2(1+B/A)/M1 \) the following expressions are obtained for the phase errors of the first and second harmonic (in radians) in a single interlacing period:

\[
p1 = \left(\frac{x}{n}\right) \left[ (Q-R) \sin f_1 + \sin 2f_1 + R \sin 3f_1 \right]
\]
p2 = (x/2Rn) [ sin f_1 + Q sin 2f_1 + sin 3f_1 + R sin 4f_1 ]

The error of the weighted phase is \( pw = w*p1 + (1-w)*p2 \). From this I find that the RMS phase errors (in radians) on frame level are:

\[
\begin{align*}
d1 &= 0.25 \frac{(X/n)}{\sqrt{0.5\times \left( (Q-R)^2 + 1 + R^2 \right)}} \\
d2 &= 0.25 \frac{(X/2Rn)}{\sqrt{0.5\times (2 + Q^2 + R^2)}} \\
dw &= 0.25 \frac{(X/n)}{\sqrt{0.5\times \left( (w(Q-R)+(1-w)/2R)^2 + (w+(1-w)Q/2R)^2 + (wR+(1-w)/2R)^2 + ((1-w)/2)^2 \right)}}
\end{align*}
\]

Here, \( X \) is the RMS value of \( x \) (apparently, \( x \) is not constant). The factor 0.25 arises because \( p1 \) and \( p2 \) refer to a single interlacing period, whereas \( d1, d2, dw \) are for a whole frame (16 interlacing periods). (Note that \( n \) is still the number of samples per interlacing period.) These RMS errors should be compared with the photonstatistical error, which is (for the first harmonic):

\[
e = 0.25 \frac{\sqrt{(2/(n-1))(B+A)}}{(A*M1)}
\]

if the first sample is discarded, or

\[
e' = 0.25 \frac{\sqrt{(2/n)(B+A)}}{(A*M1)}
\]

if the first sample is retained.

Retaining the first sample thus gives a total error of

\[
e'' = \sqrt{(e')^2 + d1^2}
\]

to be compared with \( e \) if the sample is discarded. The table below gives a numerical comparison of the various quantities as a function of magnitude. The numbers \( n \) follow from the target time versus magnitude and the distribution of stars on magnitude; for the intensities I have assumed \( B = 0.042 \) (50 Hz) and \( A = 1.667 \) (2000 Hz) at \( Hp = 9 \). \( M1 = 0.74 \) and \( M2 = 0.27 \) were assumed for the modulation coefficients. According to NDAC/LO/134, \( x = 0.084 +/- 0.042 \), from which \( X = 0.094 \).

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Explanations:

\( Hp \) = Hipparcos magnitude  
\( n \) = number of samples per star in an interlacing period  
\( d1 \) = RMS error on 1st harmonic phase [mas] caused by first sample  
\( d2 \) = RMS error on 2nd harmonic phase [mas] caused by first sample
3. Analysis of the 19 bright transits

The transits in NDAC/LO134 were reduced with and without the first sample, and the phase differences and average formal standard errors computed. Results:

average and rms difference, 1st harmonic:
\[ <d_{p1}> = 0.18 \text{ mas} \quad (d_{p1})\text{rms} = 0.39 \text{ mas} \]

average and rms difference, 2nd harmonic:
\[ <d_{p2}> = 0.22 \text{ mas} \quad (d_{p2})\text{rms} = 0.57 \text{ mas} \]

average photonstatistical error including first sample: 0.909 mas
average photonstatistical error excluding first sample: 0.945 mas
estimated total error with first sample included: 0.989 mas

A bias of the order of 0.2 mas is indicated, but its reality is not very certain. Otherwise, the numbers agree rather well with the analytical model for \( H_p = 3 \). Extrapolating to \( H_p = 5 \) we should find around the same rms differences, but the photonstatistical errors become 2.28 mas with and 2.37 without the first sample. The estimated total error with the sample included is 2.31 mas, so we conclude again that it might be advantageous to keep the sample for \( H_p > 5 \).

4. Conclusions

Retaining the first sample for stars fainter than \( H_p = 5 \) would reduce the random errors on these stars by up to 1.8%. However, the existence of a possible bias (of the order of 0.2 mas) needs to be more closely examined e.g. from a full great-circle reduction.