HIPPARCOS OBSERVATIONS OF MINOR PLANETS AND NATURAL SATELLITES: NDAC TREATMENT AND THE EFFECTS OF PHASE, SHAPE AND ALBEDO VARIATION

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ABSTRACT. The treatment of solar-system objects in the processing of Hipparcos data by the Northern Data Analysis Consortium (NDAC) is described. In order to mimic the treatment of ordinary stars as far as possible, the object is regarded as a different star (with very large proper motion) each time it passes across the field of view. The image centroids are significantly displaced (up to 40 mas) due to the phase, non-sphericity, and albedo variegation. The possible correction of these effects is discussed. It is proposed to include also the natural satellites J2 Europa and S6 Titan in the observing programme.

1. INTRODUCTION

Observation of solar-system objects by Hipparcos provides a direct connexion between the kinematically defined inertial frame, obtained by linking Hipparcos stars to quasars, and a dynamical reference frame (Ref. 1). In principle it permits to study solar system dynamics in a reference frame established by completely independent means. Although such studies are in practice severely hampered by the short mission length, they constitute nevertheless a unique check of the reductions and the kinematical frame, since a consistent solution of orbits etc would not be possible in a distorted or rotating reference frame.

Few objects are small enough (diameter < 1") and yet sufficiently bright (m_v < 12.5) to be profitably observed with Hipparcos. 63 minor planets have been proposed (Ref. 2), some of which may be too faint for actual observation. In addition, the two moons Europa and Titan may be observed (Section 5), tying the motions of Jupiter and Saturn accurately to the Hipparcos system. All of these objects show very significant displacements of their photocentre due to the oblique illumination (phase), and probably also from shape irregularity and albedo variegation (spottedness). A good understanding, modulation and correction of these effects is essential in order to benefit from the full potential of the observations.

2. OBSERVING CONDITIONS

Table 1 summarizes some characteristics of the the observing geometry relevant for the 63 proposed asteroids. The constant revolving angle of 43° constrains the phase angles and earth distances as functions of the distance from the sun (assumed = a); observation is not possible

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Table 1. Ranges and median values for some quantities of interest for the observation of minor planets

<table>
<thead>
<tr>
<th>quantity</th>
<th>min</th>
<th>max</th>
<th>median</th>
</tr>
</thead>
<tbody>
<tr>
<td>a = semi-major axis</td>
<td>1.5</td>
<td>3.4</td>
<td>2.64</td>
</tr>
<tr>
<td>A = distance from earth</td>
<td>0.6</td>
<td>4.0</td>
<td>2.44</td>
</tr>
<tr>
<td>i = phase angle</td>
<td>12</td>
<td>43</td>
<td>19 deg</td>
</tr>
<tr>
<td>ψ = apparent angular velocity</td>
<td>6</td>
<td>23</td>
<td>10 mas s⁻¹</td>
</tr>
<tr>
<td>ω = apparent angular acceleration</td>
<td>0</td>
<td>0.5</td>
<td>0.1 μas s⁻²</td>
</tr>
<tr>
<td>R = linear radius (Ref. 3)</td>
<td>10</td>
<td>512</td>
<td>80 km</td>
</tr>
<tr>
<td>ρ = angular radius</td>
<td>10</td>
<td>350</td>
<td>45 mas</td>
</tr>
<tr>
<td>Δ = phase displacement</td>
<td>0.8</td>
<td>40</td>
<td>5.5 mas</td>
</tr>
</tbody>
</table>

near opposition. The 'proper motion' at mean quadrature is ~ 41/a¹.⁵
mas s⁻¹, which cannot be neglected even within a single observing frame (2.133... s). The upper limit for the angular acceleration is given by the acceleration in geostationary orbit (0.2 m s⁻²) divided by the minimum A. The non-linearity of the motion is negligible (< 0.1 mas) over time intervals up to 1 min, such as a FOV passage, but not for the several hours of a great-circle reduction. This is the basis for regarding the planet as a 'different' star each time it appears in the FOV, represented by a mean position and a constant proper motion.

3. MINOR PLANETS IN THE NDAC PROCESSING SCHEME

3.1. Determination of grid coordinates

As part of the initial preparation of the data received from ESA, a unique object identification number (ID) is assigned to each asteroid entering the FOV. The numbering system is somewhat different from that of the programme stars, where ID is simply the sequential number (from 1 to about 115,000) in the Input Catalogue. E.g., ID = 10⁸i+j could be used to designate the jth FOV passage of the ith solar-system object. That the object is not a star is signified by ID > 10⁸. The 'new' object is entered in a catalogue with the ID number, a reference time half-way through the FOV passage, the magnitude, colour, coordinate direction and proper motion calculated for the FOV reference time (T_FOV) according to the current ephemeris.

The subsequent processing of IDT photon counts is the same as for the stars, except in the following three respects: (1) a different routine is called for calculating the proper direction; (2) the data are not used for calibration of OTF parameters (basically M₂/M₁ and the relative phase of the second harmonic); (3) the grid coordinate is defined only by the phase of the first harmonic.
3.2. The great-circle reduction

This process starts by defining the time interval (up to 12 hours) of the data set to be reduced (called a 'set'), and selecting a reference great circle (RGC) and an RGC reference time. The relevant observations (grid coordinates) are scanned in order to set up a temporary catalogue of the 2000 or so objects included in the set. This contains the object ID number, a calculated abscissa and ordinate at the RGC reference time, the time derivative of the abscissa, and the colour of the object. The abscissa, ordinate, and abscissa derivative are initially calculated from current parameters in the main working catalogue, and only the abscissa is updated by the least-squares process.

The use of a single RGC reference time for the entire set might be a problem for minor planets due to the possibly non-linear motion in the interval (up to 6 hours) between the RGC reference time (T_{RGC}) and the time of observation (T_{FOV}). This is avoided by taking

\[ a_{RGC} = a_{FOV} + (T_{RGC} - T_{FOV}) \cdot (da/dT)_{FOV} \]  

(i.e., a linear extrapolation of the motion in the FOV passage, using a fixed da/dT according to the ephemerides) as the abscissa to be adjusted. After adjustment, the 'observed' abscissa at T_{FOV} is recovered by subtracting the proper-motion term in (1).

3.3. Output

At the end of the NDAC processing, the following data will be assembled for each FOV crossing of a minor planet:

- planet number and sequential number for FOV crossing;
- RGC pole (R.A. and Decl. in J2000 system);
- time of observation (T_{FOV});
- 'observed' abscissa, corrected for the RGC zero point error (as determined by the sphere reconstitution), but not for the phase effect etc;
- estimated abscissa correction due to phase effect etc;
- estimated precision of the observed abscissa;
- approximate ordinate at the time of observation.

The abscissa is reckoned from the intersection of the RGC with the equator J2000, in positive direction as seen from the RGC pole. It may be wrong by a (small) integer multiple of the mean grid period (nominally 1.208 arcsec) due to slit ambiguity. The ordinate is the angular distance from the RGC, reckoned positive towards the RGC pole.
4. THE PHASE EFFECT AND RELATED PROBLEMS

The resolution of the Hipparcos telescope is about 0.3". For smaller objects we find (cf. 4.4) that the normal data processing (designed for point objects) gives the position of the photocentre, i.e. the barycentre of the image brightness distribution. It is in fact appropriate, when analyzing optical observations of an unresolved planet, to treat it as a point source at the three-dimensional photocentre,

\[ P = \frac{\iint I(\mu, \phi) \frac{\mathbf{r}}{\sigma} \, d\sigma}{\iint I(\mu, \phi) \, d\sigma} \quad (2) \]

\( I = \) specific intensity of light reflected towards the observer; \( \mathbf{r} = \) position vector of the surface element \( d\sigma \); \( \mu d\sigma = \) surface element projected on the sky. The vector \( \mathbf{d} = p(T-t) - \mathbf{h}(T) \), where \( T \) is the time of observation and \( \mathbf{h} \) the position of Hipparcos, defines the coordinate direction \( \langle \mathbf{d} \rangle \) to the image photocentre as well as the mean light time \( t = \frac{|\mathbf{d}|}{c} \) from different parts of the planetary surface.

4.1. The photocentre of a homogeneously scattering sphere

Consider a spherical body with homogeneous surface characterized by the scattering function \( S \). For incident solar flux \( sF \), the specific intensity of the scattered light is (Ref. 4)

\[ I(\mu, \phi) = \frac{(F/s) S(\mu, \phi; \mu_0, \phi_0)}{(3) \quad (3)} \]

\( \mu_0, \mu = \) cosines of the angles of incidence and reflection; \( \phi_0, \phi = \) azimuths of the incident and reflected beams. \( S \) depends on the azimuths only through \( |\phi-\phi| \) or \( \cos(\phi-\phi) \). Moreover, the reciprocity principle (Ref. 5) demands that \( S(\mu_0, \phi_0; \mu, \phi) = S(\mu, \phi; \mu_0, \phi_0) \).

The integrations in (2) will be made in spherical coordinates \( \alpha, \beta \) defined as follows. Let \( \mathbf{e} \) and \( \mathbf{g} \) be unit vectors from the asteroid towards the observer (Earth) and Sun. At non-zero phase angle \( i \) these define a coordinate triad \([\mathbf{x}, \mathbf{y}, \mathbf{z}]\) of unit vectors

\[ \mathbf{x} = \langle \mathbf{e} + \mathbf{g} \rangle, \quad \mathbf{z} = \langle \mathbf{e} \times \mathbf{g} \rangle, \quad \mathbf{y} = \mathbf{z} \times \mathbf{x} \quad (4) \]

\( \langle \rangle \) denotes vector length normalization. Let \( \mathbf{n} \) be the outward-directed unit vector normal to the surface element. \( \alpha, \beta \) are then defined by

\[ \mathbf{n} = x \cos \beta \cos \alpha + y \cos \beta \sin \alpha + z \sin \beta \quad (5) \]

We shall henceforth write the scattering function \( S(\alpha, \beta, i) \), since

\[ \mu = \mathbf{e} \cdot \mathbf{n} = \cos \beta \cos(\alpha+i), \quad \mu_0 = \mathbf{s} \cdot \mathbf{n} = \cos \beta \cos(\alpha-i), \]

\[ \cos(\phi-\phi_0) = (\cos i - \mu \mu_0)(1-\mu^2)(1-\mu_0^2)^{-1} \quad (6) \]
The reciprocity principle implies that \( S \) is an even function of \( \alpha \) and \( \beta \) (i.e., independent of their sign). The area of integration is

\[
|\alpha| < \frac{i}{2} - i, \quad |\beta| < \frac{i}{2}
\]

and the surface element \( d\sigma = R^2 \cos \beta \, d\beta \, d\alpha \). With \( \mathbf{n} = \mathbf{n}R \) we find

\[
P = \frac{\iint S(\alpha, \beta, i) \, n \, \cos \beta \, d\beta \, d\alpha}{\iint S(\alpha, \beta, i) \, \cos \beta \, d\beta \, d\alpha}
\]

But the \( y \) and \( z \) components of \( n \) are odd functions of \( \alpha \) and \( \beta \), while \( S \) and \( \cos \beta \) are even. Hence \( p_y = p_z = 0 \), and

\[
\frac{p_x}{R} = C(i) \equiv \frac{\iint S(\alpha, \beta, i) \, \cos^2 \beta \, \cos \alpha \, d\beta \, d\alpha}{\iint S(\alpha, \beta, i) \, \cos \beta \, d\beta \, d\alpha}
\]

From the assumptions (a) spherical body, (b) homogeneous surface, and (c) scattering in accordance with the reciprocity principle, we have thus shown that the photocentre \( p \) is located on the phase angle bisector \( \chi \) and displaced from the centre of the sphere by a fraction \( C(i) \) of its radius as given by (9). The apparent displacement for a spherical body of angular radius \( \rho \) is

\[
\phi = C(i) \rho \sin(i/2)
\]

The atmosphereless bodies of the solar system are generally thought to have light-scattering properties similar to the Moon. An often-used approximation is the Hapke-Irvine formula (Refs. 6-7),

\[
S(\alpha, \beta, i) = f(i) \frac{\mu \mu_0}{(\mu + \mu_0)}
\]

where \( f(i) \) is a function of no interest in our problem. The analytical expression for \( C(i) \) is given in Ref. 8, eqn (27); for \( 0.25 \leq i \leq 0.45 \) rad it is adequately represented by

\[
C(i) = 0.659 + 0.09 i
\]

The much more refined model by Lumme and Bowell (Refs. 9-11) yields, by numerical integration of (9), the approximate result

\[
C(i) = 0.709 + 0.10 i \quad (0.25 \leq i \leq 0.45)
\]

for the L-B parameters \( D = 0.37, \rho = 1.17, \sigma = -0.09, \) and \( \Delta_0 = 0.15 \) to 0.60. These are representative of most asteroids, whether of taxonomic type C (\( \Delta_0 \approx 0.2 \)) or S and M (\( \Delta_0 \approx 0.5 \); Ref. 10). The rather extreme parameters of J2 Europa (\( \sigma = 0.1, \Delta_0 = 0.99 \)) would make \( C(i) \) some 2-3 \% larger. Thus, a simple relation like (13) probably describes the phase displacement of any atmosphereless, spherical and homogeneous planet or satellite to within a few per cent. \( C(i) \) will be slightly larger for the more Lambert-like, cloud-covered bodies like S6 Titan.
4.2. The photocentre of a homogeneously scattering ellipsoid

The major features of asteroid lightcurves are generally thought to be caused by cross-sectional variations of a rotating, irregularly shaped body (Ref. 12). A popular model for lightcurve interpretation is a triaxial ellipsoid with semiaxis lengths \( A \geq B \geq C \). Generalization of (8) is straightforward. Let \([a \ b \ c]\) be the triad of unit vectors along the principal axes of the ellipsoid. Introducing the tensor \( E = a a' A^2 + b b' B^2 + c c' C^2 \) such that the surface is defined by \( x' E^{-1} x = 1 \), we have for the surface element with unit normal vector \( \mathbf{n} \)

\[
\mathbf{E} = \mathbf{n}, \quad u = (\mathbf{n}' \mathbf{E}^{-1} \mathbf{n})^{-\frac{1}{2}}, \quad \sigma = A^2 B^2 C^2 u^4 \cos \beta \, d\beta \, da \tag{14}
\]

from which [with integration limits as in (7)]

\[
\mathbf{E} = \frac{\iint S(a, \beta, i) \mathbf{E} \, \mathbf{n} \, u^5 \cos \beta \, d\beta \, da}{\iint S(a, \beta, i) \, u^4 \cos \beta \, d\beta \, da} \tag{15}
\]

Given the orientation of \([a \ b \ c]\) with respect to \([x \ y \ z]\), the coordinates of \( \mathbf{p} \) in the latter triad are easily computed by numerical integration. The photocentre is in general not on the \( x \) axis. Fig. 2 shows an example of the lightcurve (top) and photocentric displacement (bottom; curve A) of an ellipsoid rotating about the \( c \) axis and with the sun and observer in the plane normal to the rotation axis.

4.3. Influence of albedo variations

By lightcurve analysis alone, it is formally impossible to separate uniquely the effects of shape irregularities from those of albedo variegation (Ref. 13). The belief that light variations are chiefly due to the shape is based on the double periodicity of most lightcurves and indirect evidence of the comparative homogeneity of the surfaces. The basic ambiguity remains, however. Consider for example the two bodies depicted in Fig. 1: (A) is an ellipsoid with homogeneous surface, (B) a sphere with a bright 'equator' and dark 'poles', rotating about an axis in the 'equator'. The bodies produce virtually identical lightcurves (Fig. 2, top), but the photocentric displacements are quite different: the amplitude for the sphere (Fig. 2, curve B) is seven times larger than for the ellipsoid (curve A), and the maxima occur at very different rotational phases. We might conclude from this (admittedly contrived) example that photocentric corrections derived from shape parameters alone are, at best, uncertain; in the worst case, they could be quite fictitious.

4.4. Interaction of a resolved image with the modulating grid

Let \( \mathbf{k} \) be a unit vector in the scanning direction, i.e. normal to the slits and the line of sight. Relative to a point object, the light modulation by a spherical planet is given by the complex coefficients
Fig. 1. Two hypothetical bodies that would produce nearly identical rotational lightcurves: (A) triaxial ellipsoid with homogeneous surface (axial ratios 1.12:0.89:0.72); (B) sphere with albedo varying as \(1+2\sin^2\psi\), where \(\psi\) is the angle from the symmetry axis. The surfaces scatter according to (11) and are seen at phase angle \(i = 20^\circ\). \(\mathbf{e}, \mathbf{s}\) = directions to the observer and the sun; \(\lambda\) = rotational phase.

Fig. 2. Top: lightcurve produced by the bodies in Fig. 1. Bottom: displacement of the photocentre for the ellipsoid (curve A), the sphere with heterogeneous albedo (B), and a homogeneous sphere (constant displacement, dashed line C).
\[
g_m = \iint S(\alpha, \beta, 1) \exp[i m (2 \pi / s) \mathbf{k}' \mathbf{q}] \cos \beta \, d\beta \, d\alpha
\]

(16)

with \( j = \sqrt{-1} \), \( \rho \) = angular radius, \( s \) = grid period, and \( m = 0, 1, 2, 3 \) for the different harmonics (only \( m = 0 \) and \( 1 \) are considered here, cf. 3.1). If \( 2\pi / s \) is not too large, we may introduce \( G \) and \( H \) such that

\[
|g_1| = G \, g_0, \quad (s/2\pi) \arg(g_1) = H \, C(i) \, \rho \, \sin(i/2) \, \mathbf{k}' \mathbf{q}
\]

(17)

where \( \mathbf{q} = \mathbf{z} \times \mathbf{e} \) is a unit vector along the intensity equator. \( G \) is the 'modulation degradation' factor to be applied on \( M_1 \), while \( H \) expresses an 'exaggeration' of the phase displacement of the photocentre. It is easily verified from (16) that \( G, H \to 1 \) as \( 2\pi / s \to 0 \), demonstrating that the photocentre is effectively observed for unresolved objects. The behaviour of \( G \) and \( H \) for moderately large images is shown in Fig. 3. It appears that \( 2\pi / s \leq 2.5 \quad (\rho \leq 0.5" ) \) is the practical size limit for observation on the main grid: beyond that the modulation is much too degraded (increasing the photonstatistical error), and the phase exaggeration effect makes a reliable phase correction difficult.

![Fig. 3. Modulation degradation factor (G) and phase exaggeration factor (H) for a spherical planet scattering as (11) and seen at phase angle \( i = 20^\circ \); (a) for scans along the intensity equator \( (\mathbf{k}' \mathbf{q} = 1) \), and (b) for scans in the perpendicular direction \( (\mathbf{k}' \mathbf{q} = 0) \).](image)

5. OBSERVABILITY OF NATURAL SATELLITES

Among the natural satellites, only the Galilean moons (J1-4) and Titan (S6) are brighter than \( m_H = 10 \) at mean quadrature. Only Europa (J2) and Titan satisfy the size criterion of Section 4.4, however, having angular radii in the range 0.35 to 0.45". Other factors which need to be considered are the presence of scattered light from the parent planet and the possible modulation of that light by the main grid.
The degree of modulation for the light of Jupiter or Saturn could be estimated from (16) with $2\pi P/s \approx q \sim 100$ and 45, respectively. A conservative estimate is obtained by considering a uniformly bright circular disk (no limb darkening), yielding $G = \left[2J_1(q)/q\right] \sim 1.6q^{-2.5}$ or $< 0.5 \%$ for both planets. The scattered light from the parent planet is thus virtually non-modulated, causing an enhanced but uniform background. It is reasonable to demand that this should not exceed the average intensity of the moon. This requires a minimum separation of about 100" between the planet and the moon, at which distance the IPOV attenuation equals their magnitude difference (7.9 mag in both cases). Since the orbital radii fall in the range 160 to 210", the requirement is satisfied during about 60 % of the orbits.

To summarize, it appears that Europa and Titan are indeed observable on the main Hipparcos grid, and should be included in the observing programme. The enhanced background and the degraded modulation together increase the photonstatistical mean error of the phase determination roughly by a factor 3, increasing the 'effective' magnitudes at mean quadrature to about $m_H = 8.4$ and 11.3, respectively.

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