DEFINITION OF REFERENCE FRAMES FOR THE HIPPARCOS INSTRUMENT:

Relations between NDAC, FAST, and MATRA reference frames
(L. Lindegren, 1986 Oct 20)

A. The NDAC instrument system $[x \, y \, z]$

In Figure 1, let A and A' be the projections on the celestial sphere of the apex of the chevron centre line for the active starmapper. The instantaneous scanning circle (ISC) is the great circle through A and A'. Now let C and C' be the projections of the central slit on the main grid, and O, O' their intersections with the ISC. The instrument system $[x \, y \, z]$ is then given by the three unit vectors

$x = \text{bisector of the acute angle } OO'$;

$z = \text{pole of the ISC (in the hemisphere containing the sun)}$;

$y = z \times x$ to complete the right-handed system.

Remark: If NDAC and FAST agree on using the same central slit on the main grid, then our definitions of ISC and $x$ are virtually identical. Possible differences may derive from the definition of the image (or slit) centre (e.g., weighting of the harmonics) and should be on the mas level.

B. NDAC field coordinates $(w, z)$

For an arbitrary celestial direction, given by the unit vector $u$, we may compute the direction cosines with respect to the instrument system,

$u_x = x'y$, $u_y = y'u$, $u_z = z'u$

(the prime ' denotes scalar multiplication). We then define the field index

$f = \text{sign}(u_y) = \begin{cases} +1 & \text{for the "preceding" field} \\ -1 & \text{for the "following" field} \end{cases}$

and the field coordinates

$w = u_y \cos(\gamma_0) - f u_x \sin(\gamma_0)$

$z = u_z$

where $\gamma_0$ is a fixed number ("NDAC conventional basic angle"), close to the actual basic angle. (This value will be chosen once and for all after the initial in-orbit calibration of the basic angle.)
Remark: The nominal relation between \((w, z)\) and the MATRA field coordinates \((\eta, \zeta)\) is

\[
\begin{align*}
    w &= \cos \zeta \sin (\eta - \eta_c) \\
    z &= \sin \zeta
\end{align*}
\]

where \(\eta_c\) is the field coordinate for the centre slit \((\eta_c = \pm 0.604\) arcsec). However, the actual relation will depend e.g. on the orientation of the grid. Similarly, the nominal relation between \((w, z)\) and the MATRA grid coordinates \((g^*, h^*)\) is

\[
\begin{align*}
    w &= -g^*/F \\
    z &= -h^*/F
\end{align*}
\]

where \(F\) is the focal length in the same unit as \(g^*, h^*\). Again, the actual relation will be more complicated (see below).

C. NDAC grid coordinate \(\bar{c}\)

\(\bar{c}\) is a one-dimensional coordinate normal to the slits on the main grid. It cannot be extended beyond the main grid and it has no equivalent in the perpendicular direction. The scale of \(\bar{c}\) is set by the condition that \(\bar{c}\) is an integer at the centre of a slit on the main grid. (Actually, medium and small scale irregularities are not included, so that the true slit centre is locally given by \(\bar{c} + \Delta c = \) integer, where \(\Delta c\) represents the sum of the MSI and SSI.) The slit centre is defined by the maximum of the first harmonic of the IDT signal. The origin is defined by \(\bar{c} = 0\) on the designated central slit. \(\bar{c}\) increases in the direction of image motion, if the satellite spins in the nominal sense (i.e., positive rotation about \(z\)). Thus, \(\bar{c}\) and \(w\) increase in opposite directions.

In the great-circle reduction, the following relation between \(\bar{c}\) and \((w, z)\) is assumed (upper/lower sign refers to preceding/following field):

\[
\begin{align*}
    \bar{c} &= \pm h_{00} + (g_{10} \pm h_{10})w + (g_{01} \pm h_{01})z + (g_{20} \pm h_{20})w^2 + (g_{11} \pm h_{11})wz \\
    & \quad + (g_{02} \pm h_{02})z^2 + \left[ \pm d_{00} + (c_{10} \pm d_{10})w + (c_{01} \pm d_{01})z \right] (B-V - 0.5)
\end{align*}
\]

The nominal transformation coefficients are:

\[
\begin{align*}
    g_{10} &= -F/(\text{grid period}) = -(1400\) mm)/(0.0082\) mm) = -170731.7073... \\
    h_{00} &= \frac{1}{2} g_{10} (\gamma - \gamma_0)
\end{align*}
\]

where \(\gamma\) is the actual basic angle; all other \(g_{ij}, h_{ij}, c_{ij}, d_{ij} = 0\).
D. Relation to FAST (G, H)

(Based on the definition given in the FAST/CERGA Calibration Document, Version 1, July 1986; but with the direction of the H axis reversed.)

Assuming that FAST and NDAC use the same central slit, then (G = 0, H = 0) corresponds to (G₀ = 0, z = 0). Near this origin, G changes by 1.208 gas ("grid arcsec") for each slit (when G changes by 1). Remembering that (G, H) are angles, while (w, z) are direction cosines, and that they increase in opposite directions, one can derive the following first-order relations:

\[ w = \cos H \sin \left[ \frac{h₀₀}{g₁₀} + \frac{1}{g₁₀ + h₁₀} \cdot \frac{G}{1.208 \text{ gas}} \right] \]

\[ z = \sin \left[ \frac{1}{g₁₀} \cdot \frac{H}{1.208 \text{ gas}} \right] \]

or

\[ H = \left[ g₁₀ \cdot \arcsin(z) \right] \times 1.208 \text{ gas} \]

\[ G = \left[ (g₁₀ ± h₁₀) \cdot \arcsin(w/\cos H) ± h₀₀ \right] \times 1.208 \text{ gas} \]

E. Relation to the physical grid coordinates (G*, H*)

G* and H* are measured in mm relative to the grid marks M1 and M2. From laboratory measurements we know e.g. the scanfield geometry and the location of blemishes in these coordinates. In order to obtain the field coordinates (w, z) of the blemishes etc (for which the required accuracy is 1 arcsec), a linear relation should suffice:

\[ G^* = -F(w - w₀) - F\phi (z - z₀) \]

\[ H^* = F\phi (w - w₀) - F(z - z₀) \]

where \((w₀, z₀)\) is the position of the origin of \((G^*, H^*)\), \(F\) is the focal length, and \(\phi\) the grid rotation error about the optical axis. The transformation constants are found to be:

\[ F = \text{(average grid period in G*)} \times |g₁₀| \]

\[ \phi = \frac{g₀₁}{g₁₀} \]

\[ w₀ = \frac{G^*}{F} \]

\[ z₀ = \frac{(H^* + \phi G^*)}{F} \]

where \(G^*_c\) is the coordinate of the central slit and \((G^*_A, H^*_A)\) the coordinates of the starmapper apex.
FIGURE 1. Definition of the instantaneous scanning circle (ISC) and the $x$ axis of the instrument coordinate triad $[x \ y \ z]$. $A, A'$ = projections of the active starmapper apex; $C, C'$ = projections of the central slit $(n_c)$ of the main grid; $O, O'$ = intersection points on the ISC; $x$ = midway between $O$ and $O'$. 