CONVENTIONS AND REFERENCE SYSTEMS USED BY NDAC

by L Lindegren

IMPORTANT NOTICE:
This paper gives my understanding of some basic definitions, units, and conventions used for the exchange of data between NDAC institutes and between NDAC and other parties of the Hipparcos project. Since a complete understanding and agreement on these matters is essential to our work, I urge all persons concerned with software development and simulation within NDAC to scrutinize this paper and examine your own work in relation to it. Please inform me of any errors or misconceptions that you may discover in this document; if you disagree on some item, or if you have found it necessary to deviate from its conventions. Thank you.

1. Time
The basic time-scale used by NDAC is denoted T and is defined as the number of SI seconds (and fractions thereof) elapsed since the astronomical epoch J1988.0. This is defined as the instant

\[
\begin{align*}
J1988.0 & = 1988 \text{ Jan } 1^d 12^h 00^m 00^s .000000 \text{ TDT } \\
& = 1988 \text{ Jan } 1^d 11^h 59^m 27^s .816000 \text{ TAI } \\
& = 1988 \text{ Jan } 1^d 11^h 59^m 03^s .816000 \text{ UTC } \tag{1c}
\end{align*}
\]

The Terrestrial Dynamical Time (TDT, formerly Ephemeris Time) and other astronomical time-scales are explained in The Astronomical Almanac, Section B. Actually, the difference between atomic time and coordinated universal time, \( \Delta AT = TAI - UTC \), is not yet known for the beginning of 1988; the time given in brackets in (1c) is based on the predicted value \( \Delta AT(J1988) = 24^s \). I shall use \( \Delta AT_0 \) to denote the actual value of \( \Delta AT \) at this epoch. Thus, \( J1988.0 = 1988 \text{ Jan } 1^d 11^h 59^m 27^s .816000 - \Delta AT_0 \text{ UTC.} \) \( \Delta AT_0 \) will probably be known by mid-1987.
T has been chosen because it is a uniform, continuous scale closely representing the proper time of a geocentric observer; the unit is the SI second and the origin simply related to the standard astronomical epoch J2000.0. The relationship between T and some other time-scales of interest is shown in Table 1.

T will be the independent variable in all ephemerides (sun, earth, minor planets, geocentric satellite, orbital double stars) as well as in the continuous representation of the attitude angles. Frame mid-times, starmapper transit times, set reference times and other observation epochs are expressed directly in T.

The subroutines 'sun' and 'earth' (Lindegren, 1984 June 14, 8) use T as input argument for the barycentric ephemerides of the sun and earth. Ephemerides for the minor planets to be supplied by INCA are expected to be given on the time scale Y = julian years from J2000.0, from which

\[ T = 86400 \times 365.25 \times (Y + 12.0) \] (2)

The geocentric satellite ephemeris supplied by ESOC (DDID, 6.7) is given versus Modified Julian Date (defined as the number of days from 1950 Jan 1^d 0^h UT). The exact relation between MJD and T is not yet clear. It would appear from the DDID that MJD is just another way of expressing UTC, in which case we have

\[ \text{THGTIM} = 86400 \times (\text{MJD} - 13514.0) \] (to be confirmed) (3)

since THGTIM is UTC reckoned from 1987 Jan 1^d 0^h UTC (not counting leap seconds). The ground station time expressed as T is then obtained as

\[ T = \text{THGTIM} - 31579167^s \times 0.816000 + \Delta AT \] (4)

where the appropriate value for \( \Delta AT \) is given by the integer variable CATTAL in the Data Catalogue File (DDID, 5.6).

For calculating the frame mid-times, we can use the UTC of the first sample in the record (observation frame) given by THGTIM in the telemetry header. Further corrections are needed for the time difference between the time tag and frame mid-time (\( \Delta T_4 \)) and the propagation delay from satellite to ground station, \( \tau = \left| \tau_{\text{Sat}} - \tau_{\text{GS}} \right| / c \).
\( T \text{ (mid-frame)} = \text{THGTIM} - 31579167.816000 + \Delta AT + \Delta T_4 - \tau \) \hfill (5)

It is not clear from the DDID whether THGTIM refers to the beginning, middle, or end of the first sample in the record; hence there is still some uncertainty about \( \Delta T_4 \). Assuming that THGTIM refers to the beginning of the first sample, \( \Delta T_4 = 32/30 \) s.

The nominal scanning law \( (v_N, \varepsilon_N, \Omega_N) \) and the longitude of the nominal sun \( (\lambda_s) \) is defined in terms of the time-scale \( d \) (the time in units of 86400 SI seconds from 1988 Jan 1 \( 1^d 12^h 00^m 00^s \) UTC). This is given by

\[
d = \frac{(T - 32.184 - \Delta AT_0)}{86400} \hfill (6)
\]

Note that \( v_N, \varepsilon_N, \Omega_N \) are needed to interpret the RTAD Euler angles supplied by ESOC, and that \( \lambda_s \) is needed to interpret the NDAC heliotropic attitude angles \( v, \varepsilon, \Omega \).

2. **Celestial reference systems**

The equatorial system defined by the mean equator and equinox of the epoch J2000.0 is used for all object catalogues and ephemerides (including the geocentric satellite orbit) and all internal calculations using celestial coordinates (starmapper updates etc). Input data, such as the satellite ephemeris from ESOC, which is not in this system, must be converted to equatorial J2000 before they are being used. (One exception might be the calculation of the delay \( \tau \) in (5), which is more conveniently done in the system of the mean equinox of date.)

The orbit data provided by ESOC (DDID, 6.7) refer to the mean equator and equinox of date (that is of the reference time of the orbit interval, TREF). Conversion of the rectangular coordinates from this system to the standard J2000 system could use the transformation matrix in Table 2.

3. **Instrument systems**

Two different systems are used to express the instantaneous position of a stellar image with respect to the Hipparcos instrument: the grid coordinate \( (G) \) and the field coordinates \( (w, z) \).
3.1. The grid coordinate

The (smoothed) grid coordinate $\bar{G}$ is defined by the large-scale pattern of slits on the main grid, i.e. after an imaginary removal of all medium and small scale irregularities. (The MSI and SSI are defined in such a way that they do not contain any distortion component that could be represented by a second-degree polynomial in the two coordinates of the grid surface. Thus the separation between large scale distortion and MSI+SSI is unique.) On this 'smoothed' grid, imagine a consecutive numbering of the slits \( n = 1, 2, \ldots, 2688 \), with \( n \) increasing in the direction of image motion for the nominal spin sense. At the instant when the star image is at the centre of slit \( n \) (as defined by a maximum of the first modulation harmonic), the grid coordinate is

$$\bar{G} = n - n_c$$

(7)

where \( n_c \) is the number of the designated 'centre slit'. (The value \( n_c \) is TBD by agreement with FAST; the candidate values are \( n_c = 1344 \) and \( 1345 \).) When the image is not at a slit centre, then $\bar{G}$ is defined by interpolation.

Thus $\bar{G}$ is a continuous, one-dimensional coordinate for the position of a star image with respect to the large-scale slit pattern; it increases roughly from $-1344$ to $+1344$ as the star image moves across the field (nominal spin sense), and $\bar{G} = 0$ at the centre slit of the main grid.

3.2. The field coordinates

The field coordinates \((w, z)\) and the field index \((f)\) are defined in relation to the instrument coordinate triad \([x \ y \ z]\). This in turn is defined by the projections of the active starmapper and the centre slit of the main grid onto the celestial sphere (Figure 1). Let A, A' be the two projections of the active starmapper apex (see 5.1). The great circle through A and A' is called the instantaneous scanning circle (ISC). Let C, C' be the two projections of the centre slit of the main grid, and 0, 0' their intersections with the ISC. Then

(i) \(x\) is the point on the ISC midway between 0 and 0';

(ii) \(z\) is the pole of the ISC on the hemisphere containing the sun (so that \(z\) is about 43º from the sun); and

(iii) \(y = z \times x\) to complete the right-handed triad.
The orientation of \([x \ y \ z]\) in the equatorial J2000 system is given by the attitude (as specified, for instance, by the four angles \(\lambda, \nu, \xi, \Omega\)). Thus, knowing the attitude and the equatorial direction to an object, we may compute its direction cosines with respect to the instrument triad, \(u_x, u_y, u_z\). The field index and field coordinates are then defined as

\[
f = \text{sign}(u_y) \tag{8a}
\]

\[
w = u_y \cos(\gamma_0) - f u_x \sin(\gamma_0) \tag{8b}
\]

\[
z = u_z
\]

where \(\gamma_0\) is a fixed, agreed-upon angle close to the actual basic angle (\(\gamma\)). The value of \(\gamma_0\) (the NDAC conventional basic angle) will be fixed after the first in-orbit calibration of \(\gamma\), and should then preferably not be changed.

If the satellite spins in the nominal sense (positive about \(z\)), then \(f = 1\) for the preceding field and \(f = -1\) for the following field. During a FOV passage, the \(w\) coordinate of a star image decreases from about \(\sin(0.45^\circ)\) to \(-\sin(0.45^\circ)\), while \(z\) is approximately constant somewhere between the same limits.

3.3. **Relation between grid and field coordinates**

The transformation from \((f, w, z)\) to \(\vec{G}\) is assumed to be of the following form, depending also on the colour of the object (B-V):

\[
\vec{G} = g_{10} w + g_{01} z + g_{20} w^2 + g_{11} wz + g_{02} z^2 + \\
+ (h_{00} + h_{10} w + h_{01} z + h_{20} w^2 + h_{11} wz + h_{02} z^2) f + \\
+ \left[ (c_{00} + c_{10} w + c_{01} z) + (d_{00} + d_{10} w + d_{01} z) f \right] (B-V - 0.5) \tag{9}
\]

The Set Solution should have the capability to determine all transformation coefficients \(g_{mn}, h_{mn}, c_{mn}, d_{mn}\), except \(c_{00}\), which may be determined in Step 2/3. (\(c_{00}\) cannot be determined on a great circle.) The nominal values of the coefficients are:

\[
g_{10} = -(1400 \ \text{mm})/(8.2 \ \mu\text{m}) \approx -170731.7073 \tag{10a}
\]

(all other \(g_{mn}, h_{mn}, c_{mn}, d_{mn} = 0\)) \tag{10b}
3.4. Scanfield indices

A convention for numbering the scanfields by two integer indices has been proposed by Vagh (letter of 1985 Nov 12). The longitudinal index IG runs from 1 to 168 in the direction of increasing $\ddot{G}$, while the transverse index IH runs from 1 to 46 in the direction opposite to $z$. For all of our purposes, the large-scale distortion can be neglected when calculating scanfield indices. These can then be expressed directly in terms of $(w, z)$, e.g. as

\[ IG = \text{NINT}(0.5 \times (1 + 168 \times (1 - w/w_{\text{max}}))) \]  \hspace{1cm} (11a)

\[ IH = \text{NINT}(0.5 \times (1 + 46 \times (1 - z/z_{\text{max}}))) \]  \hspace{1cm} (11b)

where $w_{\text{max}} = z_{\text{max}} = \sin(0.45^\circ)$ and NINT() is the nearest-integer function.

4. IDT signal model

The IDT preprocessing and location estimator produce estimates of the IDT signal parameters $\beta_1$ to $\beta_5$, defined by the following expression for the expected number of counts in the $\lambda$:th IDT sample:

\[ E(N_{\lambda}) = \beta_1 + \beta_2 \left[ \cos(H_{\lambda} + \beta_3) + \beta_4 \cos 2(H_{\lambda} + \beta_3) + \beta_5 \sin 2(H_{\lambda} + \beta_3) \right] \]  \hspace{1cm} (12)

Here $H_{\lambda}$ is a reference phase for each sample defined by

\[ H_{\lambda} = 2\pi \left[ \ddot{G}_{\lambda} - \ddot{G}_{\text{frame}} + \Delta G \right] \hspace{1cm} \text{(modulo } 2\pi) \]  \hspace{1cm} (13)

with $\ddot{G}_{\lambda}$ = (smoothed) grid coordinate of the image at the mid-time of sample $\lambda$; $\ddot{G}_{\text{frame}}$ = grid coordinate at frame mid-time (= boundary between the 1280th and 1281th sample of the frame); and $\Delta G =$ MSI + SSI expressed in units of the slit period. In practice $\ddot{G}_{\lambda} - \ddot{G}_{\text{frame}}$ may be computed as $(T_{\lambda} - T_{\text{frame}}) \ddot{G}$, and $\Delta G$ may be given as a look-up table versus IG and IH. Note that, for the nominal spin sense, $\ddot{G} > 0$ and $H_{\lambda}$ is increasing with time.

From the above definition it follows that $\beta_3/2\pi$ is the fractional part of the smoothed grid coordinate at frame mid-time: $\ddot{G}_{\text{frame}} = \beta_3/2\pi + \text{integer}$. $\beta_1$ and $\beta_2$ are expressed in counts/sample, $\beta_3$ in radians, and $\beta_4$, $\beta_5$ are dimensionless.
The estimation of $\beta_1$ to $\beta_5$ from the expression above is based on those samples in a frame ($\ell = 1$ to 2560) belonging to a specific star. The observation strategy assigns samples to the different stars in lots of 8 samples, known as 'slots'. The first slot in a frame covers sample $\ell = 1$ to 8, the second slot covers $\ell = 9$ to 16, etc; the 320th and last slot in the frame covers $\ell = 2553$ to 2560. Let $s = 1 + \text{INT}[(\ell-1)/8]$ be a numbering of the slots in a frame, and $s(1), s(2), \ldots, s(n)$ the slots assigned to a specific star (where $n$, the number of slots, is $1 \leq n \leq 320$). The mean slot number of the frame observation of that star is defined as the integer

$$m_{\text{slot}} = \text{INT}\left[\frac{1}{n} \sum_{i=1}^{n} s(i)\right] \tag{14}$$

Clearly $1 \leq m_{\text{slot}} \leq 320$. Note that a star observed symmetrically about the frame mid-time will have $m_{\text{slot}} = 160$. The number of accepted samples is normally

$$n_{\text{acc}} = 8m_{\text{slot}} \sqrt{n} \tag{15a}$$

However, there should be at least the option to skip the first sample on a given star in each interleaving period, in which case we would have

$$n_{\text{acc}} = 8m_{\text{slot}} - n_{\text{ilp}} \tag{15b}$$

where $n_{\text{ilp}}$ is the number of interleaving periods containing the star in question ($1 \leq n_{\text{ilp}} \leq 16$; $n_{\text{ilp}} = 16$ for 'fully observable stars').

5. Starmapper signal model

5.1. The starmapper geometry

The nominal layout of the starmappers in relation to the main grid is shown in Figure 2. The two starmappers on either side of the main grid are designated SM1 and SM2 and may be distinguished by an index $h = 1$ (for SM1, $w > 0$) or $h = 2$ (SM2, $w < 0$). Which of the starmappers is actually used ('active') will be specified in the Data Catalogue File (DDID, 5.6, OFF = 80). Each starmapper contains two groups of four slits: the vertical group ($g = 1$) and the chevron group ($g = 2$). The position of a slit group is defined by its centre line, i.e. the nominal barycentre of the four slits. The location of each slit centre with respect to the centre line
is indicated by the magnified insert in Figure 2 (for g = 2 of SM1). The relative positions are the same for the vertical group of SM1, while the signs must be reversed for the vertical and chevron groups of SM2. Thus, the two slits closest together in a group are always on the side facing the main grid.

The starmapper geometry is described in field coordinates by means of functions

\[ w = w_{fgh}^*(z) \]  \hspace{1cm} (16)

for all combinations of indices f, g, and h. The following piecewise polynomial representations are believed to suffice:

\[ w_{f1h}^*(z) = f_{1h}^* + w_{10h}^* + w_{11h}^*z + w_{12h}^*z^2 \]  \hspace{1cm} (g = 1) \hspace{1cm} (17a)

\[ w_{f2h}^*(z) = f_{2h}^* + w_{20h}^* + \left\{ \begin{array}{l} w_{21h}^*z + w_{22h}^*z^2 \\ \end{array} \right\} \hspace{1cm} (g = 2) \hspace{1cm} (17b) \]

where \( f_{1h}^*, w_{10h}^*, w_{11h}^*, w_{12h}^*, f_{2h}^*, w_{20h}^*, w_{21h}^*, w_{22h}^* \) are effectively constant throughout the mission. The nominal values, calculated from the dimensions in Figure 1 and the nominal focal length of 1400 mm, are displayed in Table 3.

5.2. **Signal parameters**

The starmapper preprocessing estimates the three parameters \( A^*, B^*, T^* \) in the following expression for the expected number of counts in the \( \ell \)-th sample of the starmapper signal:

\[ E(N_{\ell}^*) = B^* + A^* S[(T_{\ell}^* - T^*)u] \]  \hspace{1cm} (18)

Here, \( T_{\ell} \) is the mid-time of the \( \ell \)-th sample (on the time scale \( T \)), \( u \) is the effective image velocity relative to the group centre line (\( u \approx \dot{u} \) for the vertical group, \( u \approx \dot{u} \pm \ddot{z} \) for the chevron group), and \( S \) is a known starmapper response function with origin at the group centre line. Thus, \( B^* \) and \( A^* \) are expressed in counts/sample and \( T^* \) is the transit time across the group centre line expressed on the time scale \( T \). Note that \( \dot{u} < 0 \) if the satellite spins in the nominal sense (decreasing \( w \).
The group response function $S$ shall be assumed to be a superposition of four identical but displaced 'single-slit response functions', SSR:

$$S(u) = \sum_{i=1}^{4} \text{SSR}(u - u_i)$$

(19)

with the following nominal displacements (Figure 2):

$$
\begin{align*}
  u_1 &= 0.12415/1400 & u_1 &= 0.10505/1400 \\
  u_2 &= 0.04775/1400 & u_2 &= 0.06685/1400 \\
  u_3 &= -0.06685/1400 & u_3 &= -0.04775/1400 \\
  u_4 &= -0.10505/1400 & u_4 &= -0.12415/1400 \\
\end{align*}
$$

(20)

The SSR will depend on the group index (it is wider for the chevron than for the vertical slits) and on the photometric channel ($B_T$ or $V_T$), and possibly on other effects. It is in general asymmetric, at least for the chevron slits. However, in order that the transit time ($T^*$) and stellar intensity ($A^*$) shall be uniquely defined and comparable between the different groups and channels, the following two rules define the origin and amplitude of the SSR:

(i) the origin is located at the median point of the SSR,

$$
\int_{-\infty}^{0} \text{SSR}(u) \, du = \int_{0}^{\infty} \text{SSR}(u) \, du
$$

(21a)

(ii) the total area of the SSR equals the nominal slit width,

$$
\int_{-\infty}^{\infty} \text{SSR}(u) \, du = 0.0062/1400
$$

(21b)

Condition (i) means that the transit time, in case of an asymmetric SSR, is defined by the median of the light curve. (ii) means that $A^*$ equals the stellar count rate (in counts/sample) that would be obtained with an infinitely wide slit.

Separate estimates of $B^*$, $A^*$ and $T^*$ are obtained for the two channels ($B_T$ and $V_T$). The final estimate of $T^*$ is obtained by combination of these two estimates or possibly by analysis of the combined signal.
FIGURE 1. Definition of the instantaneous scanning circle (ISC) and the $\mathbf{x}$ axis of the instrument coordinate triad $[\mathbf{x}, \mathbf{y}, \mathbf{z}]$. $A, A' =$ projections of the active starmapper apex; $C, C' =$ projections of the central slit $(n_c)$ of the main grid; $O, O' =$ intersection points on the ISC; $\mathbf{x} =$ midway between $O$ and $O'$.

FIGURE 3. The actual starmapper geometry is parametrized by means of pieces of second-degree polynomials $w = w^*_f g (z)$, depending on the field $(f)$, slit group $(g)$, and which starmapper is active $(h = 1$ shown).
FIGURE 2. Nominal grid geometry (derived from the Grid calibration plan, Issue 3, 1985 June 3). The numbers give the nominal coordinates in mm of selected slits on the main grid and of the centre lines on the starmapper. They are measured relative to the symmetry lines (dashed) in the tangential plane of the spherical surface (projection normal to the plane). The magnified portions of the chevron grid give the positions of the starmapper slits relative to the centre line. The directions in which the various coordinates increase are indicated by the compass card. Note that the axes for w, z do not coincide with the lines of symmetry, and that the origin of G is at slit number n_c = 1344 or 1345 (TBD).
<table>
<thead>
<tr>
<th>Epoch</th>
<th>Significance of epoch</th>
<th>JED</th>
<th>TAI atomic time</th>
<th>UTC universal time</th>
<th>THGTIM ground station</th>
<th>d days from UTC88</th>
<th>T NDAC time</th>
</tr>
</thead>
<tbody>
<tr>
<td>UTC87*</td>
<td>origin for the</td>
<td>2446796.5003725+AT(UTC87)/86400</td>
<td>1987 Jan 01 00h 00m+00s+ΔAT(UTC87)</td>
<td>1987 Jan 01 00h 00m+00s</td>
<td>0.000</td>
<td>-365.5000000+ΔAT(UTC87)−ΔAT 0+ΔAT(UTC87)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ground station</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>time THGTIM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J1988</td>
<td>origin for the</td>
<td>2447162.0</td>
<td>1988 Jan 01 11h 59m+27.816s</td>
<td>1988 Jan 01 11h 59m+27.816s−ΔAT 0</td>
<td>31579167.816−32.184+ΔAT 0</td>
<td>0.000</td>
<td>-31579167.816+ΔAT(UTC87)</td>
</tr>
<tr>
<td></td>
<td>NDAC time T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[31579143.8166]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UTC88*</td>
<td>origin for the</td>
<td>2447162.0003725+ΔAT 0/86400</td>
<td>1988 Jan 01 12h 00m+00s+ΔAT 0</td>
<td>1988 Jan 01 12h 00m+00s</td>
<td>31579200.000</td>
<td>0.0000000</td>
<td>32.184+ΔAT 0[36.184]</td>
</tr>
<tr>
<td></td>
<td>argument d in</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>nominal scanning law</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J1990</td>
<td>reference time for</td>
<td>2447892.5</td>
<td>1989 Dec 31 23h 59m+27.816s</td>
<td>1989 Dec 31 23h 59m+27.816−ΔAT(J1990)</td>
<td>94694367.816−730.4996275</td>
<td>63115200.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>astronomical positions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J2000</td>
<td>reference time for the</td>
<td>2451545.0</td>
<td>2000 Jan 01 11h 59m+27.816s</td>
<td>2000 Jan 01 11h 59m+27.816−ΔAT(J2000)</td>
<td>410270367.816−4382.9996275</td>
<td>378691200.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>equator and ecliptic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(equinox)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Not a standard designation
TABLE 2. Conversion of rectangular coordinates from the mean equinox of date to the standard equinox of J2000.0.

Let \( x_T, y_T, z_T \) be rectangular equatorial coordinates referred to the mean equinox of the date \( T \). To obtain coordinates \( x_{J2000}, y_{J2000}, z_{J2000} \) referred to the mean equinox of the standard epoch J2000.0, perform the matrix multiplication

\[
\begin{bmatrix}
  x_{J2000} \\
  y_{J2000} \\
  z_{J2000}
\end{bmatrix} =
\begin{bmatrix}
  r_{11} & r_{12} & r_{13} \\
  r_{21} & r_{22} & r_{23} \\
  r_{31} & r_{32} & r_{33}
\end{bmatrix}
\begin{bmatrix}
  x_T \\
  y_T \\
  z_T
\end{bmatrix}
\]

where the matrix elements are given as functions of \( T \) according to:

\[
\begin{align*}
  r_{11} &= 0.999997737090 + (1188858 \times t - 118879 \times t^2 + 1 \times t^3) \times 10^{-12} \\
  r_{21} &= 0.002236102257 + (-447206023 \times t - 2974 \times t^2 + 17 \times t^3) \times 10^{-12} \\
  r_{31} &= 0.000971737090 + (-194351169 \times t + 712 \times t^2 + 7 \times t^3) \times 10^{-12} \\
  r_{12} &= -r_{21} \\
  r_{22} &= 0.999997499921 + (999997 \times t - 99992 \times t^2 + 4 \times t^3) \times 10^{-12} \\
  r_{32} &= -0.000001086435 + (434568 \times t - 434555 \times t^2 + 1 \times t^3) \times 10^{-12} \\
  r_{13} &= -r_{31} \\
  r_{23} &= -0.000001086472 + (434591 \times t - 43459 \times t^2 + 1 \times t^3) \times 10^{-12} \\
  r_{33} &= 0.999999527863 + (1888857 \times t - 18887 \times t^2 + 2 \times t^3) \times 10^{-12}
\end{align*}
\]

Here \( t \) is the time from J1990.0 in units of 2 julian years (730.5 days), or

\[ t = T/63115200 - 1 \]

The expressions are based on Table 5.1-5.2 in *Vectorial Astrometry* (Murray, 1983).
TABLE 3. Nominal parameters for the starmapper geometry [eqn (17)] based on the dimensions in Figure 2 and the nominal focal length 1400 mm.

A. If the central slit of the main grid is $n_c = 1344$:

<table>
<thead>
<tr>
<th>parameter</th>
<th>SM1 ($h = 1$)</th>
<th>SM2 ($h = 2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$w_{10h}$</td>
<td>0.0093756786</td>
<td>-0.0093815357</td>
</tr>
<tr>
<td>$w_{11h}$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$w_{12h}$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$w_{20h}$</td>
<td>0.0154498214</td>
<td>-0.0154556786</td>
</tr>
<tr>
<td>$w_{21h}^+$</td>
<td>-1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$w_{21h}^-$</td>
<td>1.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>$w_{22h}$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

B. If the central slit of the main grid is $n_c = 1345$:

<table>
<thead>
<tr>
<th>parameter</th>
<th>SM1 ($h = 1$)</th>
<th>SM2 ($h = 2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$w_{10h}$</td>
<td>0.0093815357</td>
<td>-0.0093756786</td>
</tr>
<tr>
<td>$w_{11h}$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$w_{12h}$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$w_{20h}$</td>
<td>0.0154556786</td>
<td>-0.0154498214</td>
</tr>
<tr>
<td>$w_{21h}^+$</td>
<td>-1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$w_{21h}^-$</td>
<td>1.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>$w_{22h}$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$w_{22h}$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>