HIPPARCOS reductions for multiple stars, Ib.
by S. Söderhjelm

This note reports some further work related to that reported in NDAC/LO/066. Mainly, the weighting-scheme is improved, and some studies are made with changed modulation-coefficients.

1. The weighting

The BISOL program described in NDAC/LO/066 ("Paper Ia") used the "observed" m.e-increase in SAMP to adjust the weights for the different Fourier-parameters $b_i$. Such "cheating" is not possible in the real reductions, and it is necessary to define a better procedure.

The key idea is to see that instead of the true stellar instensities $I_k$, we observe $f(r_k)I_k$, where $f(r)$ is the sensitivity profile, and $r_k$ the effective distances from the centre of the IFOV. The pointing errors (defined by $\sigma_c$, $\sigma_F$ and $\sigma_s$, cf. Paper Ia) make $r_k$ variable, and we get an extra variance in the measures. As pointed out in Paper Ia, this gives a "wing" variance.

$$\text{var}_w_k = I_k^2 \left| \frac{df}{dr} \right|^2 (\sigma_f^2 + \sigma_s^2) \tag{1}$$

when $I_k$ falls on the steep wings of $f(r)$. In order to fit the observations, we must realize that there is also some variation for small $r$. With the $f(r)$ given in eq. (Ia;2), this "peak" variance may be approximated by

$$\text{var}_p_k = 2e-6 I_k^2 (\sigma_{eff}/I_k)^{4.9} \tag{2}$$

where

$$\sigma_{eff} = \sigma_c^2 + \sigma_F^2 + \sigma_s^2 \tag{3}$$
The expressions (Ia;12) give "ideal" $b_i$'s. The above variances should then have different influence according to the different phases for the two components. A detailed (phase-dependent) weighting is theoretically possible, but in practice it gives poor results. The main reason is probably that the phase-errors (cf. eqs. Ia;3,6,7) are neglected. A detailed weighting gives too high weights to some observations that are really disturbed by these effects.

As a better alternative, I have used the following scheme. For each set of Fourier-coefficients, I compute the variances

$$\text{var}_k = \max(\text{var}_w, \text{var}_p)$$

(4)

for each of the stars. The extra variances $V_i$ for each Fourier-component $b_i$ are then taken as the phase-averaged values

$$V_1 = \text{var}_1 + \text{var}_2$$

$$V_2 = V_3 = \frac{1}{2} M_1^2 V_1$$

$$V_4 = V_5 = \frac{1}{2} M_2^2 V_1$$

(5)

and the effective weights are simply

$$w_i = (y_i + V_i)^{-1}$$

(6)

where $y_i$ is the variance given by the Fourier fitting routine. These new weights resemble rather closely the "faked" ones used before, but they are now calculable from the known IFOV-profile and $\sigma_{\text{eff}}$-value.

The main contribution to $\sigma_{\text{eff}}$ comes from $\sigma_F$, and I have studied the effect of using a different $\sigma_F$ from the true one (as used in SAMP). It turns out that too low a $\sigma_F$ gives poor results, while a higher value may often reduce the final mean errors. From a 90-binary survey (series A-E as in Paper Ia, separations 2-36"), a $\sigma_F=3''5$ instead of the nominal 2''5 was found to give mostly smaller astrometric rms-values. The effect is largest (10-15%) for the "difficult" separations (10-25%), and when there is a sensible magnitude-difference. (For an equal-magnitude pair, the weighting acts symmetrically and has very little effect.) All in all, the higher $\sigma_F$ reduced the astrometric mean errors by some 3%, and this $\sigma_F^{\text{sol}} \approx 1.4 \sigma_F^{\text{true}}$. 
will be used henceforth in the weighting.

As noted before by Lindgren, it may also be advantageous to exclude the intensity-observations \( b_1 \) entirely. This was tested in the same runs as used to optimize \( \sigma_p \), and again there is a mean decrease of the astrometric rms-values when the \( b_1 \)-observations are excluded. The decrease is observed mostly for separations below some 12", and it is largest in series A and E (15-30%). These runs have a large colour-difference between the components, and because the modulation-coefficients in the solution program are for \( B-V=0.5 \) (cf. Ia;8), there will be some error in \( M_1 \) and \( M_2 \). Such errors were simulated in the runs described in Section 2 below, and there is good quantitative agreement as to the astrometric errors induced. As a general rule, the \( b_1 \)-observations should be excluded.

2. Influence of the modulation amplitudes

All simulations so far have been made with standard modulation-coefficients defined in (Ia;8). Because the real grid may have slightly wider than nominal slits, the influence of changed modulation-coefficients has also been investigated. Two identical simulation/solution programs have therefore been run, the difference being a 12% increase of \( I_0 \), a 5% decrease of \( M_1 \) and a 22% decrease of \( M_2 \) in the second one. (This corresponds to slit-width 3.48 instead of 3.2 microns, calculations by Lindgren.) All runs were for the standard series A-E, but the separations were either "large" (6-36") or "small" (0.3-1.0"). There were rather large statistical variations (especially for observations of a secondary component), but no general trends with magnitude or separation could be discerned. About 135 rms-ratios were obtained, giving a grand mean of 1.08±0.01. The mean rms-increase for binary star observations is thus comparable to the increase of the slit-width (1.088).

Another point that was brought up by these experiments is the necessity to have good calibrations of the modulation-coefficients. Some test solutions were made with \( M_1 \)-and/or \( M_2 \) increased by 3% from their true (SAMP) values, and with the \( b_1 \)-observations included. This had a surprisingly large effect for separations ≤ 15 arcsec, with rms-increases of 20-50%. This may be heuristically understood as a "forced" phase-error in terms of form \( M_1 \cdot \text{trig}(\phi) \), when \( I_k \) is well-determined from the \( b_1 \)-observations. (For larger separations, the \( I_k \) are "adjusted" by a faulty IPDV attenuation.) These problems are much alleviated by not using the \( b_1 \)-observations (as
already advocated above). In this case, a 3% $M_1$-increase gave a mean rms-increase of less than 5%. In particular, if both $M_1$ and $M_2$ are increased but their ratio is kept unchanged, the astrometric solution is unchanged, because the $M_1$-increases are then compensated by $I_k$-decreases. Fortunately, the OTF Calibration (following the IDT Preprocessing in the normal NDAC reductions) is expected to determine the ratio $M_2/M_1$ to a very good accuracy. This ratio is also nearly independent of the star colours, and it should not be a problem for the double star reductions.

3. Sample results for equal components

As a conclusion, Table 1 shows some results obtained (with the new weighting) for three series of equal-magnitude stars. Besides the old Ser. C (V=8.5, B-V=0.50), we have introduced Ser. F (V=10.0, B-V=0.75) and Ser. G (V=11.5, B-V=1.00). Three runs (six rms-values because of the equal components) were made in each separation-interval, $σ_F=2\"$, and the observing-time was in all cases 2+2 slots. For Ser. C, the results are close to those given in Paper Ia. For Ser. F and G, the photon statistics becomes more and more the dominant error-source, and the error-increase in the 10-25" range becomes less apparent.

Table 1. Mean (rms) astrometric deviations (mas). Subscript 1 is for the "free" solutions, subscript 2 for the "fixed" ones (common parallax and proper motions).

<table>
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<th>Sep (&quot;)</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$G_1$</th>
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<td>1.4</td>
<td>1.0</td>
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<td>3.1</td>
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<td>2.1</td>
<td>1.8</td>
<td>4.8</td>
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<td>2.6</td>
<td>2.2</td>
<td>5.4</td>
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<tr>
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<td>1.4</td>
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<td>1.4</td>
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