STATISTICAL MODELS FOR HIPPARCOS BINARIES

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Abstract:
A statistical model for the binaries in the solar neighbourhood is constructed, using schematic but plausible distribution functions for the semi-major axes, the mass-ratios and the eccentricities. The model is "calibrated" to give correct numbers of observed visual binaries and is then used to study the (closer) binaries of relevance for the HIPPARCOS mission. One main purpose is to estimate the influence of astrometric binaries observed by their photocentres. It is found that some 500 such systems (out of the 100 000 stars observed by HIPPARCOS) should show a detectably curved motion, while more than ten times more show an orbital proper motion bias greater than the HIPPARCOS accuracy 0''002/y. Also, a non-negligible fraction (1%) of all stars will be binaries with periods close to 1 year causing problems for the parallax determination. The main contribution to these figures is from faint main-sequence stars. The period-distribution for the "resolved" HIPPARCOS binaries is also obtained, and for many of them the periods are fairly short (< 100¥). Such data as these are to be used as guidelines in the construction of reduction software for the HIPPARCOS observations.

Presented at the International Conference on Astrometric Binaries, Bamberg, June 13-15, 1984
1. Introduction

The HIPPARCOS mission is primarily designed to measure parallaxes and proper motions for single stars. As is well known, however, the proportion of binaries and multiple stars is quite high, and it is important to be able to estimate in advance the kind of problems to expect in the reductions from this cause. The present paper aims to derive the statistical distribution of some important binary parameters for typical stellar groups observed by HIPPARCOS.

As is well known, the knowledge of the "true" distributions of semi-major axes, mass-ratios and eccentricities (including their variation with mass and age) for the binaries in the solar neighbourhood is very fragmentary. I have chosen therefore to make a rather coarse model, using simple, uncorrelated distribution functions. The model output is found to be rather insensitive to the input assumptions, and some useful guidelines for the HIPPARCOS reductions are finally obtained.

2. Model overview

A classical "shell"-model is constructed for whole-magnitude steps in apparent magnitude. The number of stars within $m\pm0.5$ in apparent magnitude and within $M\pm0.5$ in absolute magnitude is written as

$$N(m,M) = V_{\text{eff}}(m-M) \phi(M)$$

(1)

where $V_{\text{eff}}$ is a volume corrected for the star density decrease in the galactic $z$-direction and $\phi$ is a standard luminosity function. The stars in such an $(m-M)$-volume are all assumed to be at the distance $d_{\text{eff}}(m-M)$, and $V_{\text{eff}}$ and $d_{\text{eff}}$ depend on the star distribution model (see below). A proportion $F_{\text{bin}}$ (generally $\geq 1$, because triples are counted as 2 binaries etc.) of the stars are binaries, and these are then assumed to be distributed in semi-major axes ($a$) and mass-ratio ($q$) according to the probability density
functions $f_a(a)$ and $f_q(a,q)$. Assuming a unique mass-luminosity relation, the q-distribution can be transformed to a $\Delta m$-distribution and vice versa. The orbital planes and periastron directions are assumed to be randomly distributed, and we only have to specify finally the probability density for the eccentricities, $f_e(e)$.

In a first "calibration", we may vary the parameters of the model in order to fit some observed binary statistics. I am using for this purpose the data for visual binaries given by Heintz (1969). After the model has passed this test, it can be used to derive more interesting distributions of parameters observable by HIPPARCOS.

3. Basic assumptions and parameters

As for the magnitude system, we will assume that the effective HIPPARCOS-magnitude $m_H$ is a straight mean of Johnson's $V$ and $B$. (Unsubscripted $m$ and $M$ refer to $m_H$ and $M_H$.) The luminosity function $\phi_H(M)$ should then be some mean of $\phi_V$ and $\phi_B$, and for simplicity I have used the analytical approximation

$$
\phi_H(M) = 0.0030 \times 10^{0.05(M-1.7)} \left[ 1 + 10^{-0.22(M-1.7)} \right]^{-2.8}
$$

(2)

as can be derived from the expressions for $\phi_V$ and $\phi_B$ given by Bahcall and Soneira (1980). The values given by eq. (2) are generally within 50% of the geometric mean of some current $\phi_V$ and $\phi_B$ listed by Philip and Upgren (1983).

For the space-distribution, we assume a model galaxy with constant star density in the galactic plane and an exponential decrease with $z$. We assume also an absorbing layer with $k = 1.0$ mag/kpc uniformly filling the interval $|z| < 250$ pc. For a given scale-height $z_0$, we may then derive numerically the effective volume

$$
V_{\text{eff}}(z_0,y) = 4\pi \int_0^{\pi/2} r(y+1/2,\theta) r^2 \cos \theta \exp(-r \sin \theta / z_0) \, dr
$$

(3)
where the distance modulus is related to $r$ by

$$y = 5 \log (r/10) + k \ r \quad r < r_o$$

$$y = 5 \log (r/10) + k r_o \quad r \geq r_o$$

and where

$$r_o = 250/\sin \theta$$

The corresponding effective mean distances are given by

$$d_{\text{eff}}(z_o, y) = \frac{\int r \, dV_{\text{eff}}}{\int dV_{\text{eff}}}$$

Using the data by Ochsebien (1983) as a guide, I have used $z_o = 80$ pc for $-5.5 < M < -0.5$, $z_o = 130$ pc for $-0.5 < M < 3.5$, and $z_o = 250$ pc for $M > 3.5$.

For the mass-luminosity relation, I have used the following numerical relations

$$M_H = 2.11 - 3.81 x - 0.98 x^2 \quad 0.568 \leq x < 1.4$$

$$M_H = 5.10 - 14.34 x + 8.29 x^2 \quad -0.195 \leq x < 0.568$$

$$M_H = 4.38 - 21.72 x - 10.62 x^2 \quad -0.90 \leq x < -0.195$$

$$(M_H = 12.98 - 2.60 x \quad x < -0.90)$$

where $x$ stands for $\log (M/M_o)$. This is an almost continuous curve with (almost) continuous first derivative, and for $x > -0.90$ it conforms well to
the data for the main sequence given by Allen (1973) or Lacy (1979). (The faint extension is for numerical convenience only and has no influence on the solution.) Because the separate parts are only 2nd order in \( x \), they can be easily inverted to give the masses as function of the absolute magnitudes.

Many different distribution functions for the semi-major axes of binaries have been suggested. In order to keep the number of free parameters small, I have adopted the following form for \( f_a(u) \)

\[
  \begin{align*}
    f_a(x) &= 0 & x < -2, x > 4.5 \\
    f_a(x) &= 0.22 + 0.061 \, x & -2 \leq x < 0 \\
    f_a(x) &= 0.22 & 0 \leq x \leq 1.7 \\
    f_a(x) &= 0.35357 - 0.07857 \, x & 1.7 < x \leq 4.5
  \end{align*}
\]

where \( x \) stands for \( \log a \) (a.u.) and where \( \int_{-2}^{4.5} f_a(x) \, dx = 1 \). In its main form, the distribution (8) approximates the empirical results found by Abt (1979).

The most uncertain part in any discussion of binary statistics is the distribution of mass-ratios. Following the ideas of Abt (1979), I have first assumed different \( q \)-distributions for wide and close systems. (The dividing value of \( a, a_\text{sep} \), is very uncertain, but of the order of 10 a.u.) For \( a < a_\text{sep} \), I have used the simple power law

\[
  f_q(q) = 0.35 \, q^{-0.65}
\]

which corresponds to the approximate \( N \sim m^{0.35} \) observed by Abt in logarithmic mass-bins. For \( a > a_\text{sep} \), Abt finds a distribution of secondary masses which follows the standard ("vanRhijn") luminosity function. The problem with this is that for bright primaries almost all secondaries become
invisible faint M-dwarfs, and the binary frequency has to be increased to absurd values. As a partial remedy, one may introduce a "cut-off" magnitude difference \( D_m \) and consider only \( \Delta m (\equiv m_{\sec} - m_{\pr}) < D_m (\sim 8-10) \). This still overrepresents the large \( \Delta m:s \), as can be seen when one compares with the observations as given by Halbwachs (1983) or Herczeg (1984). For \( \alpha > a_{\text{sep}} \), I therefore finally adopted the linear \( \Delta m \)-distribution

\[
f_{\Delta m}(\Delta m) = \begin{cases} 
  (20 + 11.2 \, \Delta m) / 272 & \Delta m = 0 \text{ to } 1 \\
  (10 + 5.6 \, \Delta m) / 272 & \Delta m = 1 \text{ to } 8 
\end{cases}
\]

conforming roughly to the one given by Halbwachs. Such a \( \Delta m \)-distribution gives quite different \( q \)-distributions depending on the absolute magnitude of the primary, but it is a useful extreme assumption.

The orbital planes and the periastra are assumed to be randomly oriented, but there remains to specify the distribution of the eccentricities. Neglecting the tidally influenced period-eccentricity correlation, most studies indicate a rather uniform \( e \)-distribution. Theoretically, higher eccentricities should be more frequent, and I have used the limiting distributions

\[
f_1(e) = 1 / 0.9 \quad e = 0 \text{ to } 0.9 \\
f_2(e) = 2e / 0.81 \quad e = 0 \text{ to } 0.9
\]

4. Calibration of the model

The total star numbers (primaries) for different apparent magnitudes are quite reasonable. (We find e.g. 7040 stars with \( m_H = 6.0-7.0 \) and 50300 with \( m_H = 8.0-9.0 \).) The first interesting statistical distribution is that for apparent separations and magnitude differences for classical visual
binaries. Here we may compare with the statistics for ADS-binaries given by Heintz (1969). The present model gives the two-dimensional distribution over \( \alpha \) (angular semi-major axis) and \( \Delta \alpha \), while Heintz gives only the marginal distribution over \( \alpha \). Heintz estimates, however, a completeness limit at about

\[
0.22 \, \Delta \alpha - \log \alpha (") = 0.75
\]  

(13)

and our present data may thus be transformed for comparison with his.

The only free parameters in the model are the total binary frequency \( F_{\text{bin}} \) and the dividing value \( a_{\text{sep}} \) for the q-distributions. One reasonable model (A) has \( a_{\text{sep}} = 10 \) a.u. and \( F_{\text{bin}} = 1.0 \). Small shifts of \( a_{\text{sep}} \) have very little influence, but in order to have an extreme case, the "close" q-distribution was used for all systems \( (a_{\text{sep}} \rightarrow \infty) \). Surprisingly, the Heintz statistics could be equally well satisfied with this model (B), at \( F_{\text{bin}} = 1.8 \). (This high \( F_{\text{bin}} \) is due to the large number of small-q systems given by eq. 9.) Table 1 shows the calculated numbers of binaries and Heintz's observed numbers for three different intervals of apparent magnitude.

<table>
<thead>
<tr>
<th>m = 6-7</th>
<th>m = 7-8</th>
<th>m = 8-9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha (&quot;) )</td>
<td>obs A B</td>
<td>obs A B</td>
</tr>
<tr>
<td>(-0.5)</td>
<td>51 18 33</td>
<td>109 48 83</td>
</tr>
<tr>
<td>(-0.3)</td>
<td>65 37 70</td>
<td>119 98 170</td>
</tr>
<tr>
<td>(-0.1)</td>
<td>63 53 92</td>
<td>139 143 230</td>
</tr>
<tr>
<td>(+0.1)</td>
<td>89 71 103</td>
<td>171 189 264</td>
</tr>
<tr>
<td>(+0.3)</td>
<td>108 86 110</td>
<td>228 232 284</td>
</tr>
<tr>
<td>(+0.5)</td>
<td>96 102 110</td>
<td>233 273 283</td>
</tr>
<tr>
<td>(+0.7)</td>
<td>117 118 109</td>
<td>261 306 271</td>
</tr>
<tr>
<td>(+0.9)</td>
<td>113 126 102</td>
<td>207 330 262</td>
</tr>
<tr>
<td>(+1.1)</td>
<td>122 133 96</td>
<td>211 342 233</td>
</tr>
</tbody>
</table>
In view of the uncalibrated magnitude-scale, the agreement is as good as can be expected. The only obvious systematic trend is the low number of close systems predicted by model A.

5. Detection of binaries by HIPPARCOS

The wide variety of possible binary configurations (separations, magnitude differences, orbital motion) makes it difficult to generalize, but as a starting point we may use the data given by Lindegren (1982). For relatively fixed secondaries, he derives detection limits in the \((\text{sep}, \Delta m)\)-plane that may be realistic for the brighter pairs. (We must have a cut-off at \(\ell = 13\text{-}14\) which certainly precludes observation of systems with e.g. \(\Delta m = 4\text{-}5\) for a 12:\text{th} magnitude primary.) A conservative straight-line approximation similar to the Heintz criterion (13) gives the following conditions for possibly resolved pairs

\[
0.22 \Delta m - 1g \alpha < 1.3
\]

\[
\Delta m < 13.5 - m_{pr}
\]

Systems outside these limits are observed as single (moving) stars. (For separations greater than some 0"5, the primary is observed, otherwise the photocentre.) One primary goal for the present investigation is to estimate how many of these astrometric pairs that may be detected from their curved motions, and also how the undetected ones bias the proper motions.

For orbital periods above some 5 years, the observations of an astrometric binary will be distributed along a curved arc, as as a measure of curvature I have used the vector

\[
s = r[(t_1 + t_2)/2] - [r(t_1) + r(t_2)]/2
\]
where \( \mathbf{r} \) is the radius vector to the photocentre in units of its semi-major axis \( a_{\text{ph}} \), and the mission extends from \( t_1 \) to \( t_2 \). As a fair approximation, we may then assume that perpendicular to some direction, the deviations of the path from a straight line can be written

\[
\Delta x = -\frac{2}{3} s + 4 s(t - t^2)
\]  

(16)

where \( t_1 \) is now equal to 0 and \( t_2 = 1 \). For uniformly spaced observations, the variance of \( \Delta x \) is then \( 4s^2/45 \), and assuming the HIPPARCOS scans to be randomly oriented relative to the "reference" direction, we find an extra variance in the positions along the scan circle (the "abscissae") equal to one half of this, viz.

\[
\frac{\text{rel}}{\text{absc}} = 2 \frac{s^2}{45}
\]  

(17)

Different binaries of course give widely different \( s^2 \)-values, and it is necessary to investigate the expected distribution of \( s^2 \) as function of orbital period and eccentricity. This was done numerically for a large number of binaries (40000) of given period and eccentricity, but with uniform distributions over orientation angles and mean anomalies. The combined distributions for the two different \( e \)-distributions differ relatively little, and in each case we find a rather flat distribution with "wings" extending about 1.5 dex in \( s^2 \). With a spacing in \( \lg P \) of 0.1 dex, it is easy to interpolate a numerical \( s^2 \)-distribution at any \( P > 5' \).

For very short orbital periods, the entire orbit is covered more or less uniformly. For this ideal uniform case, and with scans in random directions, we now have something like

\[
\frac{\text{rel}}{\text{absc}} = \frac{1}{2} |\mathbf{r} - \langle \mathbf{r} \rangle|^2
\]  

(18)
The distribution of this quantity was again calculated numerically, and the only lacking part of the distribution of \( v_{\text{absc}}^{\text{rel}} \) is the difficult region with \( P \approx 1-5^\circ \). Somewhat arbitrarily, I have chosen to use the "short-\( P \)" distribution for all \( P < 2.5^\circ \), and to interpolate linearly between the \( 2.5^\circ \) and \( 5^\circ \)-distributions for a \( P \) in this range.

For the proper motions we proceed similarly. For \( P > 5^\circ \), we let

\[
P = \frac{r(t_2) - r(t_1)}{t_2 - t_1}
\]

(19)

and derive the distribution of \( p^2 \) as that of \( s^2 \) before. The short-\( P \) limit is now assumed to be zero, and the \( 2.5-5^\circ \) data are interpolated as before. (It is important to note that \( s^2 \), \( v_{\text{absc}}^{\text{rel}} \) and \( p^2 \) are so far given in units of the angular semi-major axis of the photocentre, \( a_{\text{ph}} \)).

6. The HIPPARCOS astrometric binaries

The general binary star model predicts the number of systems in bins of e.g. \( \log \alpha \), \( \log P \), \( q \) and \( \Delta m \). The semi-major axis of the photocentre is given by

\[
(\alpha_{\text{ph}}/\alpha)^* = \frac{q}{1+q} - (1 + 10^{0.4 \Delta m})^{-1}
\]

(20)

and by this equation we may transform the original multi-variate distribution to one over the primary variables \( \log \alpha_{\text{ph}} \) and \( \log P \). For each bin in this distribution, we have a distribution of \( v_{\text{absc}}^{\text{rel}} \) and \( p^2 \), and by a suitable convolution, we finally obtain univariate distributions of \( v_{\text{absc}} \) and \( v_{\text{pm}} \).

The extra abscissa variance may be detected when it exceeds the observational scatter by a certain factor. For a standard 1\% \( \chi^2 \)-criterion, the detection limit is at about \( \log v_{\text{absc}} \approx -5.0 \) (arcsec\(^2\)) for the brighter stars, increasing to maybe \( \log v_{\text{absc}} \approx -4.5 \) for an 11\:th mag primary. Table 2 gives the expected numbers of stars with \( \log v_{\text{absc}} > -5.05 \) for different apparent magnitudes and for the A and B models defined above. Postscripts l
and 2 refer to the eccentricity-distributions (11) and (12), which are seen to have little influence on these results. Altogether, relatively few stars are detectable by their curved motion, and by separating the absolute magnitudes, it may be shown that they are mostly F-K dwarfs.

Table 2. Numbers of astrometric binaries with detectably curved motion of their photocentres.

<table>
<thead>
<tr>
<th>model</th>
<th>2-3</th>
<th>3-4</th>
<th>4-5</th>
<th>5-6</th>
<th>6-7</th>
<th>7-8</th>
<th>8-9</th>
<th>9-10</th>
<th>10-11</th>
<th>2-9</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>3</td>
<td>7</td>
<td>17</td>
<td>34</td>
<td>61</td>
<td>96</td>
<td>138</td>
<td>176</td>
<td>204</td>
<td>357</td>
</tr>
<tr>
<td>A2</td>
<td>2</td>
<td>7</td>
<td>16</td>
<td>33</td>
<td>57</td>
<td>90</td>
<td>128</td>
<td>161</td>
<td>186</td>
<td>333</td>
</tr>
<tr>
<td>B1</td>
<td>5</td>
<td>13</td>
<td>31</td>
<td>62</td>
<td>109</td>
<td>172</td>
<td>248</td>
<td>316</td>
<td>367</td>
<td>642</td>
</tr>
<tr>
<td>B2</td>
<td>5</td>
<td>13</td>
<td>29</td>
<td>59</td>
<td>103</td>
<td>161</td>
<td>229</td>
<td>290</td>
<td>334</td>
<td>601</td>
</tr>
</tbody>
</table>

As for the proper motion variance \( \sqrt{v_{pm}} \) due to binaries, it affects sensibly a larger proportion of stars. Table 3 shows the percentages of stars with \( \sqrt{v_{pm}} > 0.002 \) at different apparent magnitudes. As a comparison, Lindegren (1978) obtained the round figure 10% for \( m_p = 9 \) stars (from a more limited study).

Table 3. Relative numbers of stars (%) with \( \sqrt{v_{pm}} > 0.002 \) /year.

<table>
<thead>
<tr>
<th>model</th>
<th>2-3</th>
<th>3-4</th>
<th>4-5</th>
<th>5-6</th>
<th>6-7</th>
<th>7-8</th>
<th>8-9</th>
<th>9-10</th>
<th>10-11</th>
<th>2-9</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>9.9</td>
<td>10.7</td>
<td>11.0</td>
<td>10.7</td>
<td>10.1</td>
<td>8.7</td>
<td>7.0</td>
<td>5.4</td>
<td>3.9</td>
<td>7.9</td>
</tr>
<tr>
<td>A2</td>
<td>9.5</td>
<td>10.2</td>
<td>10.4</td>
<td>10.1</td>
<td>9.4</td>
<td>8.1</td>
<td>6.5</td>
<td>4.9</td>
<td>3.6</td>
<td>7.3</td>
</tr>
<tr>
<td>B1</td>
<td>16.1</td>
<td>16.0</td>
<td>15.2</td>
<td>13.8</td>
<td>12.1</td>
<td>10.1</td>
<td>7.9</td>
<td>5.9</td>
<td>4.3</td>
<td>9.1</td>
</tr>
<tr>
<td>B2</td>
<td>15.4</td>
<td>15.2</td>
<td>14.4</td>
<td>13.0</td>
<td>11.3</td>
<td>9.3</td>
<td>7.3</td>
<td>5.5</td>
<td>3.9</td>
<td>8.4</td>
</tr>
</tbody>
</table>

It is instructive to separate the stars according to absolute magnitude. Table 4 gives the resulting percentages for apparent magnitude \( m = 8-9 \).
Table 4. Relative number of stars (%) showing various degrees of proper motion variance. Apparent magnitude interval $m = 8-9$.

<table>
<thead>
<tr>
<th>$\nu_{\mu \mu}$</th>
<th>M -1.5 to +0.5</th>
<th>+0.5 to +2.5</th>
<th>2.5 to 5.5</th>
<th>&gt; 5.5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_1$</td>
<td>$B_1$</td>
<td>$A_1$</td>
<td>$B_1$</td>
</tr>
<tr>
<td>$&gt; 0^\circ 002/\gamma$</td>
<td>0.8</td>
<td>0.9</td>
<td>6.6</td>
<td>7.1</td>
</tr>
<tr>
<td>$&gt; 0^\circ 005/\gamma$</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$&gt; 0^\circ 010/\gamma$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$&gt; 0^\circ 020/\gamma$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$&gt; 0^\circ 050/\gamma$</td>
<td>-</td>
<td>-</td>
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</tr>
</tbody>
</table>

For the faintest stars, the proportion of large "binary" proper motions is high, but of course their "true" proper motions are also higher.

From the distribution of astrometric binaries in the ($\log \alpha_{\phi}$, $\log P$)-plane we may also estimate the number of cases with P close to one year and $\alpha_{\phi} > 0^\circ 002$ where the parallaxes may become faulty. Table 5 gives these numbers for the $\log P$ interval $-0.1$ to $+0.1$. A division by luminosity shows again that the percentages are higher for fainter absolute magnitudes.

Table 5. Numbers of stars where the parallax error may exceed 50%.

<table>
<thead>
<tr>
<th>model $m$</th>
<th>2-3</th>
<th>3-4</th>
<th>4-5</th>
<th>5-6</th>
<th>6-7</th>
<th>7-8</th>
<th>8-9</th>
<th>9-10</th>
<th>10-11</th>
<th>2-9</th>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>%</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>B</td>
<td></td>
<td></td>
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<td></td>
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<td></td>
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<td></td>
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</tr>
</tbody>
</table>

All the figures in Tables 2-5 are for an assumed "main-sequence" luminosity function. As shown by Halbwachs (1984), we should expect also a rather high proportion of degenerate components to apparently single main-sequence stars, and I have made some preliminary calculations to include this in the model. Using the same a- and e-distributions as for the main-sequence binaries, and assuming that a typical degenerate component has a mass equal to $0.8 M_\odot$, I find that the figures in Tables 2-5 should be increased by 20-50% (higher values for the A model) for a 10% fraction of
degenerates. The uncertainty due to this cause is thus of the same typical magnitude as the difference between the A and B models.

7. The resolved HIPPARCOS binaries

In Section 6 we obtained the statistics for binaries observed as single stars by HIPPARCOS. The resolution criterion (14) is necessary, but not sufficient, because Lindegren's (1982) study implicitly assumed components fixed relative to each other. The present model allows us to study the period-distribution among the "tentatively resolved" pairs. There are many combinations of separations, magnitude differences, and absolute and apparent magnitudes to be considered, but in Table 6 are given some results for the "typical" m = 8-9 interval. The numbers are for all "new" (undetected by the Heintz criterion, but not necessarily by the time of the HIPPARCOS mission) binaries, regardless of A m. As expected, the close pairs also show the more rapid motion, and there will obviously be detection problems for some systems in the upper left corner of Table 6. (For apparently brighter pairs, the periods are shifted to even shorter values, aggravating the problems.)

Table 6. "Resolved" binaries according to separation and period for apparent magnitude m = 8-9.

<table>
<thead>
<tr>
<th>lg sep (&quot;)</th>
<th>-1.1</th>
<th>-0.9</th>
<th>-0.7</th>
<th>-0.5</th>
<th>-0.3</th>
<th>-0.1</th>
<th>+0.1</th>
<th>+0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>lg P(y)</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>&lt; 1</td>
<td>13</td>
<td>20</td>
<td>8</td>
<td>6</td>
<td>0</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.0-1.5</td>
<td>61</td>
<td>85</td>
<td>44</td>
<td>51</td>
<td>22</td>
<td>29</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>1.5-2.0</td>
<td>137</td>
<td>153</td>
<td>170</td>
<td>210</td>
<td>130</td>
<td>139</td>
<td>62</td>
<td>51</td>
</tr>
<tr>
<td>2.0-2.5</td>
<td>43</td>
<td>82</td>
<td>166</td>
<td>261</td>
<td>270</td>
<td>343</td>
<td>192</td>
<td>211</td>
</tr>
<tr>
<td>2.5-3.0</td>
<td>-</td>
<td>-</td>
<td>16</td>
<td>40</td>
<td>114</td>
<td>204</td>
<td>254</td>
<td>300</td>
</tr>
<tr>
<td>3.0-3.5</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>7</td>
<td>51</td>
<td>70</td>
<td>190</td>
<td>179</td>
</tr>
<tr>
<td>3.5-4.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>&gt; 4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

N: 653

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A separation according to absolute magnitude shows that again the faint nearby pairs are the problematic ones. Table 7 gives the star numbers in column 5 of Table 6 (lg sep = -0.5), and similar distributions are obtained for other separations.

Table 7. Numbers of binaries according to absolute magnitude and period for m = 8-9, separation = 0''25 - 0''40.

<table>
<thead>
<tr>
<th>M</th>
<th>-5.5 to -1.5</th>
<th>-1.5 to +0.5</th>
<th>0.5 to 2.5</th>
<th>2.5 to 5.5</th>
<th>&gt; 5.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>lg P</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>1.0-1.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.5-2.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2.0-2.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>86</td>
</tr>
<tr>
<td>2.5-3.0</td>
<td>0</td>
<td>3</td>
<td>107</td>
<td>140</td>
<td>159</td>
</tr>
<tr>
<td>3.0-3.5</td>
<td>23</td>
<td>43</td>
<td>29</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>3.5-4.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

8. Conclusions

The statistical model described above is thought to contain the general features of the "real" distribution of binaries. From the numerical results we may tentatively conclude:

1. Variation of the assumed input distributions give relatively small (less than a factor two) changes in the output. Conversely, the HIPPARCOS observations will not greatly increase our knowledge of binary statistics.

2. The Abt model, with different distributions of the mass-ratio (q) for wide and close binaries, will be difficult to distinguish from models with a single distribution.
3. Only a few hundred astrometric binaries are detectable by HIPPARCOS on account of their curved paths on the sky. Many more (of the order of 10%) will show orbital proper motions of their photocentres greater than 0\'002/year.

4. About 0.5-1% of the stars are binaries with photocentric semi-major axes greater than 0\'002 and periods in the range 0\'8 - 1\'25 where the parallax determination is influenced.

5. A large number of binaries may be resolved (observed as two separate components) by HIPPARCOS. Many of them show sensible orbital motion, however, and the reduction methods have to take this into account.

Acknowledgement: This work is supported by the Swedish Board for Space Activities.

References:


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Lindgren, L.: 1978, Colloq. on European Space Astrometry, Padova, p. 117
