Chromaticity and data reductions

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Problem

In a telex from MAC Perryman (22164/dh, 9.12.82), the data reduction consortium were asked to assess the impacts of chromaticity in regard to computational burden, accuracy degradation, stability requirements, etc. Two cases should be considered:

(a) that adequate colours are available for all programme stars;
(b) that adequate colours are available only for the prs (perhaps half of the stars, mostly brighter?), while remaining colours are essentially unknown.

Realistic chromaticity maps and expected temporal behaviour will be provided by ESTEC. The only information received thus far is MATRA’s predicted constant and variable chromaticity: \( C_c = 2.1 \) mas, \( C_v = 1.0 \) mas.

Assumptions

We shall assume that the chromaticity effect on the grid coordinate \( G \) can be expressed, for all stars and to within a fraction of a mas, in the linear form

\[
\Delta G = c_1 (X_0 \mp X_1) + c_2 (X_2 \mp X_3) + c_3 (X_4 \mp X_5)
\]  

(1)

where \( c_i \) is a constant for each star, computable from its colour, \( X_m \) are chromaticity coefficients constant for at least 24 h, and upper/lower sign refers to preceding/following field. The origin and scale of the colour index \( c_i \) should be chosen so that \( \langle c_i \rangle \approx 0 \) for the average of all programme stars, and \( \partial c / \partial \delta (B-V) \approx 1 \). As a first approximation we could assume

\[
c_i = (B-V)_i - 0.6
\]  

(2)

but a non-linear function of \( B-V \) may be more appropriate, or possibly one involving also \( U-B \) (or other colours from other systems).

Most important about the form (1) is the assumption that the entire chromaticity map simply scales with \( c_i \) for the different stars. If this is not the case, there is probably much less hope to disentangle it from other effects.
Method 1: Strict incorporation in set solution

Given an explicit model like (1), the most satisfactory way to eliminate chromatic effects is by incorporating it directly in the set solutions, and solving for $\chi_m$ exactly as for the achromatic distortion parameters. Recall that the set observation equations (001.1) or (016.6) are of the form

$$A_{ik} \Delta \chi_i + B_k \Delta \psi_k + \sum_{\ell} \eta_{ik} \zeta_{ik} (\Delta g_{ik} + \Delta h_{ik}) = (G_{obs} - G_{cat})_{ik}$$

(3)

with $i =$ star index, $k =$ frame index and $A_{ik}, B_k =$ known coefficients.

Case (a): All colours known

In this case we add to the left member of (3) the six terms

$$+ c_1 \Delta \chi_0 + c_1 \Delta \chi_1 + c_{1i} \Delta \chi_2 + c_{1i} \Delta \chi_3 + c_{1i} \Delta \chi_4 + c_{1i} \Delta \chi_5$$

(4)

Given that $c_1 \sim 0$ when averaged over observations in each quadrant of the FOV, there should be no strong correlations with the achromatic distortion parameters. The situation should in fact be more advantageous than when introducing the eight third-order achromatic distortion parameters. This was shown to have negligible impact on the abscissa solution but doubled the computing time (Petersen, 1983 Feb 28). If most of the iterations (e.g. for eliminating slit inconsistencies) can be made without instrument parameters, the net effect on computing time is of course less dramatic. From this point of view it is probably feasible to use either third-order achromatic polynomials only, or second-order achromatic plus first-order chromatic distortion; but not third-order achromatic plus first-order chromatic.

Case (b): Colours known only for a subset of stars

The colour indices $c_1$ must now be regarded as additional unknowns for remaining stars. The left member of the observation equation then consists of the terms in (3) plus (4) plus

$$+ \left[(\chi_0 + \eta_{ik} \chi_2 + \zeta_{ik} \chi_4) + (\chi_1 + \eta_{ik} \chi_3 + \zeta_{ik} \chi_5)\right] \Delta c_i$$

(5)

This approach seems to have very important reprecussions on several levels:

- From a datamatic point of view, the new term (5) is of a completely different kind than (4), since it entails one more unknown per star rather than just an augmentation of the instrument parameter list. Division of stars in two categories (known/unknown colours) is also a new concept.
- The new unknowns \( c_i \) are in principle the same in different sets (for a given star). To establish the required link (constraint) between the different sets in a rigorous way is practically impossible. A three-step procedure would be required as for the astrometric parameters, viz.: (i) determine independent colours in each set; (ii) calculate the average colour of each star from the whole mission; (iii) insert these colours in the set equations as in Case (a) above to get definitive abscissae.

- The addition of perhaps 1000 unknowns per set would certainly increase computation time by a significant factor on top of that caused by the six chromatic parameters in (4).

- Perhaps the most important consideration however is that the conditioning of the solution depends not only on the form of chromaticity, i.e. (1), but also on the actual values of \( \chi_m \), which are now part of the coefficient matrix. Certain combinations of \( \chi_m \) will in fact make the solution indeterminate. Consider for instance the (quite reasonable) case that \( \chi_0 \) is much larger than \( \chi_m \), \( m > 0 \). The set may then give a good determination of the quantities \( \Delta \dot{c}_i + \chi_0 \Delta c_i \), but separation of \( \Delta \dot{c}_i \) and \( \Delta c_i \) is virtually impossible, since it would depend entirely on the field-dependent terms in (5). In other words, the determination of \( c_i \) relies on the variation of chromaticity across the field or between preceding and following field; is the variations are small, the colour is indetermined. But \( \chi_0 \) may still be large, requiring an accurate colour to determine the abscissa.

In conclusion, Method 1 is not feasible in Case (b).

**Method 2: Residual analysis**

The set solution is performed without chromatic terms, i.e. using observation equation (3). Afterwards, residuals \( r_{ik} \) are computed for each observation and sorted according to field position, colour (if known), magnitude, etc. This material is then subjected to off-line multifactor analysis (essentially linear regression and/or binning) in order to detect possible correlations.

This method should preferably be used as a diagnostic tool rather than for actual calibration (although it is in fact proposed for the medium-scale geometric grid calibration). Among its advantages are: it is made off-line, requires negligible computation, is suitable for interactive processing, can take into consideration a large number of 'unlikely' factors, and may be to some extent parameter-free (binning) and therefore less model dependent.
The analysis is the same in Case (a) and (b), except that the material is smaller in the latter case.

The method is primarily a null test to confirm (ideally) the expected randomness of residuals and hence give confidence in the reduction model. If on the other hand significant correlations are revealed (e.g. chromaticity), it is not at all trivial how to proceed. Firstly, the derived chromaticity map may be strongly biased because of hidden correlation with other factors such as the achromatic distortion parameters. Secondly, if the derived map is reasonably correct, how can it be used to correct the abscissae even for stars of known colour? The only 'safe' way would be to correct individual observations and iterate the solution. Analysis of the new residuals should then hopefully show that the effect was indeed removed. Computationally it may however be more efficient to use the direct Method 1 (once a parametric model has been established), rather than iterating via residual analysis. In both methods the problem with unknown colours remains in Case (b).

Conclusions

The method of residual analysis should be applied as a matter of routine. It is a simple and valuable tool for monitoring the quality of data as well as reduction model, and for establishing or checking the functional form and stability of chromaticity. It may be useful also for eliminating chromaticity on stars with known colour. For this last purpose, a rigorous solution (Method 1) is of course preferred and probably computationally feasible if enough colours are known.

In Case (b) we do not see any way to eliminate chromatic effects on those stars without colours. It is even possible that they to some extent corrupt the results also for the remaining stars, although a good reduction model should prevent this from happen.

It is almost impossible to make quantitative estimates of the effects on the final results without rather detailed simulation runs. With reasonable confidence we can only state that they are probably negligible provided (i) that at least approximate colours are known for most stars; (ii) that the total chromaticity is no more than a few mas; and (iii) that most of it is constant over time periods of 24 h.