HIPPARCOS

Star Distribution Models (2)

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Summary

An analytical expression is given for the smoothed star density as function of blue magnitude and galactic coordinates (b, \( \xi \)).

The formula

A previous note, "Star Distribution Models" (1982-11-14) gave the star density as function of blue magnitude (B) and galactic latitude (b). The present note extends the formula with a term depending on galactic longitude (\( \xi \)).

Let \( N(B, b, \xi) \) be the (smoothed) number of stars per square degree at galactic coordinates (b, \( \xi \)) brighter than B. For all B < 16 and all positions we have approximately

\[
\lg[N(B, b, \xi)] = \lg[N(B, b)] - e^{0.39(B-22) \cos^2 b} -
\]

\[
- 0.1 \left[ 1 + \ln \left( 1 + e^{0.65(B-12.7)} \right) \right] \lg \left[ 1 - \frac{1.56 - 0.60 \cos b}{1 + e^{-0.78(B-9)}} \cos b \cos \xi \right] \tag{1}
\]

in which

\[
\lg[N(B, b)] = -3.65 + 0.489 B - 0.002 B^2 -
\]

\[
- (0.86 - 0.084 B + 0.007 B^2)(1.4|\sin b| - 0.4 \sin^2 b) \tag{2}
\]

gives the density averaged with respect to \( \xi \):

\[
N(B, b) = \frac{2\pi}{\pi} \int_{-\pi}^{\pi} N(B, b, \xi) d\xi \tag{3}
\]

Averaging also with respect to b (i.e. over the full sky), we have

\[
N(B) = 2^{1/2} \int_{-\pi/4}^{\pi/4} N(B, b) \cos b \, db \tag{4}
\]
which for $B < 16$ is approximated by

$$\lg N(B) = -4.08 + 0.528B - 0.005B^2$$

(5)

The densities given by (2) and (5) are about 0.02 dex smaller than in the previous note, and the coefficient for $B$ in (5) is also slightly changed to give better consistency with (2).

The general form of dependence on $B$ and $b$ was adopted from Allen (1973), but with a zero point shift in $\lg(N)$ such that the total number of stars to $B = 8.5$ agrees with counts in the CSI (Turon Lacarrue, 1980). The variations with longitude were taken from the Galaxy model by Bahcall and Soneira (1980).

Tables 1 to 3 give some comparisons with these sources.

Table 1. Comparison of (2) with Allen (1973). The table gives $\lg(N_{LL}/N_{Allen})$.

<table>
<thead>
<tr>
<th>B</th>
<th>$b = 0^\circ$</th>
<th>$5^\circ$</th>
<th>$10^\circ$</th>
<th>$20^\circ$</th>
<th>$30^\circ$</th>
<th>$40^\circ$</th>
<th>$50^\circ$</th>
<th>$60^\circ$</th>
<th>$90^\circ$</th>
<th>all sky</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>(.25)</td>
<td>(.13)</td>
<td>(-.03)</td>
<td>(.17)</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>2</td>
<td>(.06)</td>
<td>(.09)</td>
<td>(-.04)</td>
<td>(.14)</td>
<td></td>
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<tr>
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<td>.06</td>
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<td>.04</td>
<td>.04</td>
<td>.05</td>
<td>.07</td>
<td>.06</td>
<td>.04</td>
<td>.03</td>
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<td>.05</td>
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<tr>
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<td>.08</td>
<td>-.01</td>
<td>.08</td>
<td>.08</td>
<td>.07</td>
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<td>.05</td>
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<td>.12</td>
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Table 2. Comparison of (1) with Bahcall and Soneira (1980): $\lg(N_{LL}/N_{BS})$.

<table>
<thead>
<tr>
<th>B</th>
<th>$b = 20^\circ$</th>
<th>$20^\circ$</th>
<th>$20^\circ$</th>
<th>$30^\circ$</th>
<th>$30^\circ$</th>
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Table 3. Comparison of (2) and (5) with the CSI (Turton, 1980): $\log(N_{LL}/N_{CS})$.

<table>
<thead>
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<th>B</th>
<th>$0 - 5^\circ$</th>
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<th>$10 - 20^\circ$</th>
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<th>$40 - 60^\circ$</th>
<th>$60 - 90^\circ$</th>
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<td>.001</td>
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<td>.004</td>
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<td>-.007</td>
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<td>9.0</td>
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<td>9.5</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.011</td>
</tr>
</tbody>
</table>

Galactic coordinates

For convenience I give here the complete and accurate transformation between ecliptical coordinates ($\lambda$, $\beta$), referred to the standard epoch J2000.0, and galactic coordinates ($b$, $\ell$):

$$
\begin{pmatrix}
\cos b \cos \ell \\
\cos b \sin \ell \\
\sin b
\end{pmatrix} = \begin{pmatrix}
-0.548777621 & -0.9938212536 & -0.0964766664 \\
0.4941083214 & -0.1109918634 & 0.8622863637 \\
-0.8676666398 & -0.0003495777 & 0.4971463365
\end{pmatrix} \begin{pmatrix}
\cos \beta \cos \lambda \\
\cos \beta \sin \lambda \\
\sin \beta
\end{pmatrix}
$$

(6)

References


Turton Lacarrieu, C.: 1980, Preliminary remarks on the definition of a survey, Hipparcos working paper 1980 Oct 8 (Table 1 and 2)