Simulation of ambiguity solution by mesh method

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This note contains some suggestions for a preliminary test of the mesh method (Hög, 1982 Nov 12) for solving slit ambiguities in step 3.

Unknowns
For the present purpose it is adequate to work in ecliptical coordinates. For each star, the unknowns are the five differential astrometric parameters

\[ a_1 = \Delta \lambda \cos \beta \quad [0.5 \text{ to } 1.5 \text{ as }] \]
\[ a_2 = \Delta \beta \quad [0.5 \text{ to } 1.5 \text{ as }] \]
\[ a_3 = \Delta \mu_\lambda \cos \beta \quad [0.01 \text{ to } 0.05 \text{ as/yr}] \]
\[ a_4 = \Delta \mu_\beta \quad [0.01 \text{ to } 0.05 \text{ as/yr}] \]
\[ a_5 = \Delta \Pi \quad [0.01 \text{ to } 0.05 \text{ as }] \]

(corrections to the catalogue values), and the slit errors \( \eta_j \)s of each set (\( j = 1 \text{ to } \text{NSET} \)), \( s = 1.2 \text{ as} \).

For each experiment, true values of the unknowns must be chosen or (better) computed with a pseudo-random number generator. Gaussian distributions with s.d. in the ranges indicated in (1) [brackets] are suggested. Also \( \{ \eta_j \}_j^{\text{NSET}} \) are generated with s.d. 0.5 to 1.5 units.

Observation equations
One observation equation is generated per set (12 h interval). The program SCANS should be modified so as to accept only one observation per 12 h interval, even when the observing condition is satisfied in several spins. The result is the partial derivatives \( p_{jk} = \partial \eta_j / \partial a_k \), \( k = 1 \text{ to } 5 \), in set \( j \). The true value of the rhs is

\[ r_{j,\text{true}} = \Sigma_k p_{jk} a_{k,\text{true}} - \eta_{j,\text{true}} s \]

from which "observed" rhs is obtained by adding an observational error with s.d. \( \sim 0.05 \text{ to } 0.15 \text{ as} \):

\[ r_j = r_{j,\text{true}} + \sigma_{rj} * \text{GAUSS} \]
The actual observation equations are

\[ \sum_k p_{jk} a_k + \text{noise} = r_j + n_j s \quad \text{(m.e.} = \sigma_{r_j}), \quad j = 1 \text{ to NSET} \quad (4) \]

or

\[ Pa + v = r + ns \quad (5) \]

with noise covariance

\[ C = \text{Cov}(y) = \text{diag}(\ldots, \sigma_{r_j}^2, \ldots) \quad (6) \]

Solution

Let \( z = r + ns \) in (5). For given \( \{n_j\} \), the normal equations are

\[ P' C^{-1} Pa = P' C^{-1} z \quad (7) \]

with solution

\[ a = D(P' C^{-1} z), \quad D = (P' C^{-1} P)^{-1} \quad (8), (9) \]

The normalized residuals are

\[ u = C^{-1/2}(z - Pa) \quad (10) \]

and the sum of the squares of the normalized residuals is

\[ \chi^2 = u'u = (z' - a'P')C^{-1}(z - Pa) = z'C^{-1}z - a'(P'C^{-1}z) \quad (11) \]

The following procedure is proposed:
1. Set up the (NSET,5)-matrix $P$, the NSET-array $r$, and the NSET-array $c$ containing the mean errors.

2. Compute $D$, eqn (9).

3. Choose a starting approximation (mesh point), $a^{(0)} = (x, y, 0, 0, 0)'$.

4. For $j = 1$ to NSET:
   4.1. $n_j = \text{NINT}[(\sum_k p_{jk} a_k^{(0)} - r_j)/s]$  \hspace{1cm} (12)
   4.2. $z_j = r_j + n_j s$  \hspace{1cm} (13)
   4.3. Accumulate to the 5-array $P'C^{-1}z$ and the scalar $z'C^{-1}z$.

5. Solve $a$, eqn (8).

6. Compute $\chi^2$, eqn (11).

7. If $\chi^2 >$ some limit depending on NSET, goto 3 (new mesh point).