HIPPARCOS

Phase Errors of a Trapezoidal Slit Profile

L Lindegren (1982 Nov 15)
Lund Observatory
Box 1107
S-22104 LUND, Sweden

The transmittance profile of a single slit is usually taken to be rectangular, i.e. with infinitely sharp edges. In reality the edges are of course more or less fuzzy, possibly to different degrees on opposite edges, thus causing an asymmetric slit profile and different phase shifts for the two harmonics of the detector signal.

As a first approximation to such an asymmetric profile we may take the trapezoidal form defined by the four parameters (cf. Figure)

\( s = \) grid period
\( r = \) slit width (FWHM)
\( a = \) width of left edge slope
\( b = \) width of right edge slope

Thus, the transmittance \( T(x) \) for the basic period \(-\frac{1}{2}s \leq x < \frac{1}{2}s\) is

\[
T(x) = \begin{cases} 
0 & \text{for } -\frac{1}{2}s \leq x < -\frac{1}{2}(r+a) \\
\frac{1}{2}(r+a) + x/a & \text{for } -\frac{1}{2}(r+a) \leq x < -\frac{1}{2}(r-a) \\
1 & \text{for } -\frac{1}{2}(r-a) \leq x < \frac{1}{2}(r-b) \\
\frac{1}{2}(r+b) - x/b & \text{for } \frac{1}{2}(r-b) \leq x < \frac{1}{2}(r+b) \\
0 & \text{for } \frac{1}{2}(r+b) \leq x < \frac{1}{2}s
\end{cases}
\]
This can be expressed as a Fourier series,

\[ T(x) = \sum_{n=-\infty}^{\infty} c_n \exp\left(j2\pi nx/s\right) \]  

(2)

where the coefficients are computed from

\[ c_n = \frac{1}{s} \int_{-\frac{s}{2}}^{\frac{s}{2}} T(x) \exp(-j2\pi nx/s) \]  

(3)

Direct integration yields

\[ c_n = \frac{1}{2} \left( A_n + B_n \right) \frac{\sin(n\pi \delta)}{n\pi} - j \frac{1}{2} \left( A_n - B_n \right) \frac{\cos(n\pi \delta)}{n\pi} \]  

(4)

where

\[ A_n = \frac{\sin(n\pi a/s)}{n\pi a/s} \quad (= 1 \text{ for } na = 0) \]  

(5a)

\[ B_n = \frac{\sin(n\pi b/s)}{n\pi b/s} \quad (= 1 \text{ for } nb = 0) \]  

(5b)

The amplitude and displacement of the nth harmonic of the slit profile are implicit in \( c_n \) and \( c_{-n} = c_n^* \). A more easily interpreted form than (2) is however

\[ T(x) = T_0 + T_1 \cos(2\pi(x-d_1)/s) + T_2 \cos(4\pi(x-d_2)/s) + \ldots \]  

(6)

where \( T_n, d_n \) are the amplitude and displacement of the nth harmonic.

Comparing (6) and (2), and introducing (4), we find

\[ T_0 = c_0 = \delta \]  

(7a)

\[ T_n = \frac{1}{n\pi} \left[ A_n^2 - 2A_n B_n \cos(2\pi n\delta) + B_n^2 \right]^{\frac{1}{2}} \quad (n > 0) \]  

(7b)

\[ d_n = \frac{s}{2\pi n} \arctan\left( \frac{A_n - B_n}{A_n + B_n} \cot(n\pi \delta) \right) \quad (n > 0) \]  

(7c)

For perfect edges, \( a = b = 0 \), we have \( A_n = B_n = 1 \) and hence

\[ T_n^{(0)} = 2\sin(n\pi \delta)/(n\pi) \quad (n > 0) \]  

(8)

\[ d_n^{(0)} = 0 \quad (n > 0) \]  

(9)
The degradation of the amplitude due to the finite edges is therefore

\[
D_n = \frac{T_n}{T_n^{(0)}} \approx \frac{1}{2 \sin(n\pi \delta)} \left[ A_n^2 - 2A_n B_n \cos(2n\pi \delta) + B_n^2 \right]^{1/2}
\]  

(10)

Although (10) and (7c) give the effects of the edges on the harmonics of the slit profile, it is clear that the harmonics of the light modulation will suffer exactly the same degradation and displacement (neglecting grid diffraction effects).

When \(a\) and \(b\) are small, we can obtain some simpler approximations for \(D_n\) and \(d_n\). From (5) we get

\[
A_n \approx 1 - \frac{1}{6} (n\pi a/s)^2, \quad B_n \approx 1 - \frac{1}{6} (n\pi b/s)^2
\]  

(11)

which should be accurate to better than 1% for \(a, b < s/(n\pi)\). Since only \(n = 1\) and \(2\) are of interest here, we shall assume

\[
a, b < \frac{s}{2\pi}
\]  

(12)

for the approximations. Then \(A_n + B_n \approx 2\) and \(A_n - B_n \approx (n\pi/s)^2(b^2 - a^2)/6\).

If furthermore \(|\cot(n\pi \delta)| < 1\), we get from (7c)

\[
d_n \approx \frac{n\pi}{24s} \cot(n\pi \delta) (b^2 - a^2)
\]  

(13)

and from (10), with similar approximations,

\[
D_n \approx 1 - \frac{1}{12} (n\pi/s)^2 (a^2 + b^2)
\]  

(14)

It can be noted that \(d_1\) and \(d_2\) are proportional; the ratio

\[
d_2/d_1 \approx 2 \tan(\pi \delta)/\tan(2\pi \delta)
\]  

(15)

is moreover negative for \(0.25 < \delta < 0.5\), which is the relevant interval (\(d_2/d_1 \approx -5.4\) for \(\delta = 0.38\)).

More interesting than the absolute displacements \(d_n\) (which depend on how the origin \(x = 0\), or slit centre, is defined) is perhaps the relative displacement...
\[ d_2 - d_1 = \frac{\pi}{24s} \left[ 2\cot(2\pi\delta) - \cot(\pi\delta) \right] (b^2 - a^2) \]  
(16)

**Numerical Example:**

A very tentative specification for the edge sharpness was given by me in a note dated 79-06-18, viz.

\[ \Delta_{0.1}, \Delta_{0.9} < 0.25 \text{ } \mu \text{m} \]  
(17)

This corresponds roughly to \( a, b < 0.5 \text{ } \mu \text{m} \approx 0.075 \text{ arcsec } (P = 1.4 \text{ m}) \), or

\[ a^2 + b^2 < 0.01125 \text{ arcsec}^2 \]  
(18)

With \( s = 1.2 \text{ arcsec} \), eqn (14) gives

\[ D_1 > 0.9936, \quad D_2 > 0.974 \]  
(19)

If we assume 30% asymmetry, i.e. \( |a - b|/(a + b) = 0.3 \), we get

\[ |b^2 - a^2| < 0.0062 \text{ arcsec}^2 \]  
(20)

so that eqn (16) gives (for \( \delta = 0.38, s = 1.2 \text{ arcsec} \))

\[ |d_2 - d_1| < 0.0017 \text{ arcsec} = 1.7 \text{ mas} \quad (\text{N.B.: } |d_1| < 0.3 \text{ mas}) \]  
(21)

It thus appears that even a strong asymmetry (30%) is harmless as long as the edges are sharp enough not to cause serious modulation losses.