HIPPARCOS

Coefficients of Improvement — An Historical Review

L. Lindegren (1982 June 27)

The "Step 3 Coefficients of Improvement" (COI) were computed several times
during and after the Phase A Study, under various assumptions and using
(partly) independent software. The five COI $c_1 - c_5$ (for $\Delta \lambda \cos \beta$, $\Delta \beta$, $\mu_\lambda \cos \beta$,
$\mu_\beta$, and $\tau$, respectively) depend mainly on $\xi$ (revolving angle), $T$ (mission
length in years), $R$ (number of great circle scans per day), and $\phi$ (transverse
field of view). In order to compare the different determinations of $c_i$ it
is necessary to know at least approximately their dependence on $\xi$, $T$, $R$, and
$\phi$, so that the different sets of COI can be referred to identical assumptions.
The analytical derivation of $c_i$ in Höyer et al. (Astron. Astrophys. 101, 228,
1981) can be used for this purpose. The results in the interval $\xi = 20 - 50^0$
are accurately approximated by the functions

$$c_1^* = \left[ 365 \frac{RT}{\sin(\frac{\phi}{2})} 0.632 \sin \xi \right]^{-1}$$  \hspace{1cm} (1)

$$c_2^* = \left[ 365 \frac{RT}{\sin(\frac{\phi}{2})} (1 - 0.632 \sin \xi) \right]^{-1}$$  \hspace{1cm} (2)

$$c_3^* = \frac{\sqrt{12}}{T} c_1^*$$  \hspace{1cm} (3)

$$c_4^* = \frac{\sqrt{12}}{T} c_2^*$$  \hspace{1cm} (4)

$$c_5^* = \left[ 365 \frac{RT}{\sin(\frac{\phi}{2})} 0.5 \sin^2 \xi \right]^{-1}$$  \hspace{1cm} (5)

Since the derivation by Höyer et al. neglect correlations between the five
astrometric parameters, $c_i^*$ should underestimate the true COI (which can only
be computed by numerical methods). They can nevertheless be used to translate
the COI computed for one parameter set $\{\xi, T, R, \phi\}$ into COI for a standard
set $\{\xi_0, T_0, R_0, \phi_0\}$ according to

$$c_i(\xi_0, T_0, R_0, \phi_0) \cong \frac{c_i^*(\xi_0, T_0, R_0, \phi_0)}{c_i^*(\xi, T, R, \phi)} \quad c_i(\xi, T, R, \phi)$$  \hspace{1cm} (6)

The Table below summarizes results obtained during 1976-79; the last line
gives $c_i^*$ for comparison. The standard parameter set used is $\xi = 40^0$, $T = 2.5$,
$R = 12$, and $\phi = 54^\prime$. 
<table>
<thead>
<tr>
<th>Date</th>
<th>Author(s)</th>
<th>ξ</th>
<th>T</th>
<th>R</th>
<th>Φ</th>
<th>c₁</th>
<th>c₂</th>
<th>c₃</th>
<th>c₄</th>
<th>c₅</th>
<th>c₁</th>
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<td>45°</td>
<td>3.0</td>
<td>15.16</td>
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<td>Vaghi MAD 76 Fig. 9b</td>
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</table>

* from analytical theory disregarding correlations
Comments to the Table

The first two determinations (Lindegren 1976, 1977) were based on very few points on the sphere (one longitude only for 1976; two latitudes only for 1977). Still they agree surprisingly well with later determinations.

The ESOC MAD Working Paper No. 76 appears to give too small COI after the latitude averaging (e.g. Fig. 11 in MAD 76). Probably the weighting of the different latitudes is in error. Therefore the COI versus latitude were taken from Fig. 9b and the sky average computed with correct weighting (cos β). The ratio $c_3/c_1$ is somewhat higher than expected.

The COI quoted in PF616 are from the AML Report, Fig. 18 (Symmetrical Scanning), adjusted to $T = 2.5$ yr. The unsymmetrical scanning (Fig. 19) gives slightly better COI.

Fig. 5.3 in SCI(79)10 is based on MAD 76 with corrected latitude weighting and some additional adjustments on the proper motion COI to obtain the theoretical $c_3/c_1$ and $c_4/c_2$.

Table 5.3 in SCI(79)10 gives the relative mean errors versus latitude. This is obviously incorrect in that the mean errors in longitude and latitude must be roughly equal at $\beta = \pm 85^\circ$. The strong variation with latitude shown for $\Delta \lambda \cos \beta$, $\mu_\lambda \cos \beta$, and $\dot{\omega} (\pi)$ is not found in the computations quoted above. In fact I have not been able to trace the source of these numbers!