The All-Reflective Eccentric Schmidt (ARES)

L. Lindegren (1982 April 18)

Summary
The aberrations of the nominal ARS can probably be considerably reduced by displacing the two halves of the Schmidt corrector by about 4 mm in opposite directions along the z axis. This means that an 8 mm wide strip is removed from the corrector before reassembling the beam combiner.

INTRODUCTION
The large aberrations of the nominal ARS are mainly caused by the use of an inclined corrector with circular figuring. As viewed from the spherical mirror, the figuring is then elliptic with axial ratio \( \cos \theta = 0.968 \) for inclination \( \theta = 14.5^\circ \). Thus, if the corrector is designed to eliminate spherical aberration in the scanning direction (along y), there is still considerable spherical aberration in the transverse direction (z), which unfortunately causes an MTF loss also in the scanning direction.

This effect could in principle be eliminated by elliptical figuring of the corrector. A much simpler device would be to approximate the desired elliptical figuring by means of a displaced or eccentric circular figuring (Fig. 1). The required displacement is of the order of \( (\sec \theta - 1)R \approx 5 \) mm for pupil radius \( R = 0.16 \) m. This possibility is explored below by means of an approximate analytical expression for the WFE at the field centre of the eccentric Schmidt system.

WAVE-FRONT ERRORS OF THE ECCENTRIC SCHMIDT
In order to compensate for the spherical aberration of a spherical mirror of curvature radius \( R_p = 2f \), a WF retardation by

\[ D(y,z) = \gamma(y^2 + z^2)^2 - \alpha(y^2 + z^2) \] (1)
is required. Here, \( y, z \) are pupil coordinates (\( y \) along scan), while

\[
\gamma = 1/(32f^3)
\] (2)

and

\[
\alpha = d/(4f^2)
\] (3)

is related to the position of the focal surface at \( x_f = \frac{1}{2}R_p - d \).

Thus, for given focal length \( f \), \( \gamma \) is a fixed number while \( \alpha \) can be adjusted to any suitable number by refocussing. Now (1) shall be approximated by the retardation produced by the inclined eccentric corrector.

Let \( \rho \) be the radial distance along the corrector surface from its symmetry point at \( y = 0, z = -e \). The figuring will be of the form

\[
h(\rho) = Ap^2(P^2 - \rho^2)
\] (5)

where \( A \) is the asphericity coefficient and \( P \) is the radius at which the thickness \( h \) of the deposit is reduced to zero. In order that \( h \geq 0 \) over the entire corrector, we must have

\[
P \geq \max(R/\cos i, R + e)
\] (6)

where \( R \) is the radius of the (circular) pupil and \( i \) is the inclination of the corrector. The maximum deposit thickness is

\[
h_{\text{max}} = AP^4/4
\] (7)

for radius \( \rho = P/\sqrt{2} \), so one should preferably have equality in (6) for minimum \( h_{\text{max}} \).
A ray parallel to the optical axis (x) with pupil coordinates (y, z) will intersect the corrector at a radius given by

$$\rho^2 = \frac{y^2}{c^2} + (z + e)^2$$  \hspace{1cm} (8)

where \(c = \cos i\) for brevity. It will experience a retardation

$$\Delta(y,z) = -2c \chi (\rho) =$$

$$= -2cA \left( \frac{y^2}{c^2} + (z + e)^2 \right) \left[ \rho^2 - \frac{y^2}{c^2} - (z + e)^2 \right]$$  \hspace{1cm} (9)

so that the WFE is

$$W(y,z) = \Delta(y,z) - D(y,z)$$  \hspace{1cm} (10)

or

$$W(y,z) = a(y^2 + z^2) - \gamma (y^2 + z^2)^2 - \left( 2A/c^3 \right) \left[ y^2 + (z + e)^2 \right] \times$$

$$\times \left[ \rho^2 c^2 - y^2 - (z + e)^2 \right]$$  \hspace{1cm} (11)

It is seen that the coefficient of \(y^4\) is \((2A/c^3) - \gamma\), so that the "longitudinal spherical aberration" will disappear by choosing the asphericity

$$A = \frac{1}{2} \gamma c^3$$  \hspace{1cm} (12)

Putting \(\beta = a/\gamma\), we then have

$$W(y,z) = \gamma \left\{ \left[ \beta - \rho^2 c^2 - 2z^2 + 2(z + e)^2 \right] y^2 +$$

$$+ z^2 - \rho^2 c^4 (z + e)^2 - z^4 + c^4 (z + e)^2 \right\}$$  \hspace{1cm} (13)

For any given \(e\), we can make the WFE constant along any desired line \(z = z_o\) by choosing the focus such that

$$\beta = \rho^2 c^2 + 2z_o^2 - 2(z_o + e)^2 c^2$$  \hspace{1cm} (14)
TRANSVERSE ABERRATIONS

The transverse (ray) aberrations are obtained as the partial derivatives of the WFE:

\[ \eta = \frac{\partial W}{\partial y} = 2\gamma\left[ (\beta - 2r^2) - c^2(p^2 - 2p^2) \right] \]  
(15)

\[ \zeta = \frac{\partial W}{\partial z} = 2\gamma\left[ z(\beta - 2r^2) - (z+e)c^4(p^2 - 2p^2) \right] \]  
(16)

where \( r^2 = y^2 + z^2 \) and \( g^2 \) is given by (8).

RESULTS

The Concentric Schmidt (ARS)

Parameters used are:

- \( f = 1.55 \text{ m} \)
- \( R = 0.16 \text{ m} \)
- \( \varepsilon = 0.4 \) (obscuration ratio)
- \( i = 14.5^\circ \)
- \( z_O = 0 \)
- \( P = 0.202406 \text{ m} \)

The aphericity coefficient of the corrector is

\[ A = \frac{c^3}{(64f^3)} = 0.003807586 \text{ m}^{-3} \]

while the value of \( P \) corresponds to the paraxial curvature radius

\[ R_{\text{corr}} = 3205.32 \text{ m} \]

and maximum figuring depth

\[ h_{\text{max}} = AP^4/4 = 1.59766 \times 10^{-6} \text{ m} \]

The assumption \( z_O = 0 \) (i.e. constant WFE along \( y \) axis) is pure conjecture.

The resulting WFE matrix (in nm) is shown in Fig. 2a, while Fig. 3a gives the ray aberrations and Fig. 4a the distribution of rays along the \( y \) axis and along an axis at \( 45^\circ \) inclination.
The Eccentric Schmidt (ARES)

The parameters used are the same as for the ARS except

\[ e = 0.004 \text{ m} \]
\[ z_0 = 0.08 \text{ m} \]
\[ P = 0.165264 \text{ m} \]

The displacement \( e = 0.004 \text{ m} \) was found to be nearly optimum in terms of the concentration of rays both along the scan and in the 45° direction. \( z_0 \) was chosen = \( R/2 \) because this is about the most important part of the pupil for the MTF at the first grid harmonic (spatial frequency \( \approx 0.1 \text{ m} \)). \( P = R/\cos i \) was chosen in order to minimise \( h_{\max} = 7.10073 \times 10^{-7} \text{ m} \).

The WFE matrix, ray aberrations, and ray distributions are shown in Figs. 2–4b alongside the corresponding figures for the ARS.

The two systems are compared in the Table below.

<table>
<thead>
<tr>
<th></th>
<th>ARS</th>
<th>ARES</th>
</tr>
</thead>
<tbody>
<tr>
<td>focal length ( f )</td>
<td>1.55 m</td>
<td>1.55 m</td>
</tr>
<tr>
<td>pupil diameter ( R )</td>
<td>0.16 m</td>
<td>0.16 m</td>
</tr>
<tr>
<td>obscuration ( \varepsilon )</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>inclination ( i )</td>
<td>14.5 deg</td>
<td>14.5 deg</td>
</tr>
<tr>
<td>curvature radius of corrector ( R_{corr} )</td>
<td>3205 m</td>
<td>4808 m</td>
</tr>
<tr>
<td>asphericity ( A )</td>
<td>0.0038076 m⁻³</td>
<td>0.0038076 m⁻³</td>
</tr>
<tr>
<td>maximum deposit thickness ( h_{\max} )</td>
<td>1.6 ( \mu \text{m} )</td>
<td>0.7 ( \mu \text{m} )</td>
</tr>
</tbody>
</table>

Ray aberrations:

- rays within \( \pm 0.05^\circ \):
  - \( 0^\circ \): 29 %, 67 %
  - \( 45^\circ \): 26 %, 51 %
- rays within \( \pm 0.15^\circ \):
  - \( 0^\circ \): 56 %, 94 %
  - \( 45^\circ \): 60 %, 77 %

MTF losses: \( < \cos(2\pi[W(y+u,z)−W(y,z)]/\lambda) >, \lambda = 500 \text{ nm} \):

\[ u = 0.10 \text{ m} \] 0.62 0.98
\[ u = 0.20 \text{ m} \] 0.89 0.99
FIGURE 1. Principle of eccentric Schmidt:

(a) - ideal elliptical figuring of inclined corrector (axial ratio = $\cos i$)
(b) - circular figuring of the ARS
(c) - eccentric circular figuring of ARES
**WFE MATRIX IN NM FOR F, R: ANCL, E: ZD, P = .1550000E+01 .1600000E+00 .1450000E+02 .0000000E+00 .0000000E+00 .0000000E+00 .202406E+00**

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**ARS**

-4 2 -4 -14 -29 -47 -70 -97 -128 -163 -202 -245 -293 -344 -400 -460 -524
50 49 43 34 22 6 -14 -37 -63 -94 -127 -165 -206 -250 -298 -350 -405
80 78 74 66 55 42 25 3 -17 -43 -72 -104 -139 -176 -217 -261 -308
95 94 90 84 75 63 49 33 14 -8 -32 -59 -86 -120 -154 -191 -231
100 99 96 91 83 74 62 49 33 15 5 -27 -51 -78 -106 -137 -169
97 96 93 89 83 75 66 55 42 28 12 -6 -26 -47 -70 -95 -121
88 87 85 81 77 71 63 55 44 33 20 6 -9 -26 -44 -64 -85
75 74 72 70 66 62 56 49 42 33 23 12 0 -13 -27 -41 -57
60 59 58 56 53 50 46 41 35 29 22 14 5 -4 -15 -26 -37
44 43 42 40 38 35 31 27 23 18 12 6 0 -7 -19 -23
30 29 28 27 26 24 21 19 16 13 9 5 1 -3 -8 -13
17 17 17 16 15 14 13 11 10 8 6 4 1 -1 -4 -7
8 8 8 8 7 7 6 5 5 4 3 2 1 0 -2 -3
2 2 2 2 2 2 1 1 1 1 1 0 0 0 0 -1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

**FIGURE 2a.** WFE matrix (in nm) for ARS
(1 by 1 cm mesh; half pupil only)

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**WFE MATRIX IN NM FOR F, R: ANCL, E: ZD, P = .1550000E+01 .1600000E+00 .1450000E+02 .4000000E-02 .8000000E-01 .165264E+00**

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**ARES**

-4 -5 -6 -8 -10 -12 -15 -18 -20 -25 -30 -33 -37 -44 -51 -56 -69 -78 -89

**FIGURE 2b.** WFE matrix (in nm) for AREs
(1 by 1 cm mesh; half pupil only)
$\sqrt{\text{rms}} \approx 0.23 = 1.23 \mu m$

$\sqrt{\text{rms}} \approx 0.67$

$= 0.53 \mu m$

FIGURE 4a. Ray histogram for ARS

FIGURE 4b. Ray histogram for ARES

(1370 rays on 5 by 5 mm mesh over full semi-circular pupil)