HIPPARCOS / TYCHO

Optimization of Non-Periodic Star Mapper Patterns

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SUMMARY

It is shown how the combination of a non-periodic star mapper slit pattern and a numerical filter can be optimized for maximum side lobe suppression. Examples of slit patterns and filters for up to 9 slits are given.

1. FORMULATION OF PROBLEM

We shall consider a group of N identical slits positioned on a regular mesh, so that the distance between any two slits is always a multiple of the basic mesh distance (Fig. 1). The mesh distance will typically be about 3 arcsec but must be optimized with regard to the MTF and slit width. It should also be an integer multiple (m) of the distance travelled by a star image during the star mapper sample interval (vΔt = 0.25 arcsec for v = 150 arcsec/s, Δt = 1/600 s).

Any such "allowed" slit pattern can be represented by a sequence \( \{g_i\}_{i=-\infty}^{\infty} \) where \( g_i = 1 \) if there is a slit at mesh point \# i, otherwise \( g_i = 0 \). The first slit may be taken as origin for the mesh, so that \( g_0 = 1 \) and \( g_1 = 0 \) for \( i < 0 \). In order to search systematically for a "good" slit pattern, it is necessary to give an enumeration of all possible patterns. This can be done by means of the number

\[
S = \sum_{i=-\infty}^{\infty} g_i 2^i
\]

which we shall call the "signature" of the slit pattern \( \{g_i\}_{i=-\infty}^{\infty} \). With the above convention for the mesh origin, \( S \) is always an odd positive integer. Thus the search can be made from the sequence \( S = 1, 3, 5, 7, \ldots \) ad infinitum.

The pattern shown in Fig. 1 is the sequence \( \{g_0 \text{ to } g_7 = 1, 1, 0, 0, 1, 0, 0, 1\} \) with signature \( S = 10010011_2 = 147 \). For our purposes this pattern is equivalent to the retroversion, \( \{g_0 \text{ to } g_7 = 1, 0, 0, 1, 0, 0, 1, 1\} \) with \( S = 11001001_2 = 201 \). Now if \( R(.) \) signifies the bit-reversing operator, we have for instance
FIGURE 1. Example of slit pattern \( g_i \), SM response function \( f(x) \), filter coefficients \( G_i \), filter curve \( F(x) \), \( \Gamma = G \ast g \), and expected filter output \( E(y_i) \) for unit intensity star at position \( p = 0 \).
R(147) = 201 and R(201) = 147. Thus we need only examine patterns with e.g. S < R(S). For symmetric patterns R(S) = S, which case should also be studied.

It is assumed that the star mapper response function, f(x), can be expressed as the convolution of \( \{g_i\}_{i=-\infty}^{\infty} \) with the single slit response function h(x):

\[
f(x) = \sum_{i=-\infty}^{\infty} g_i h(x - mi)
\]

(2)

where \( m \) is the basic mesh distance in units of \( v \Delta t \) (Fig. 1). The star mapper photon counts \( \{n_j\} \) is assumed to be a Poisson process with expectation

\[
E(n_j) = b + a f(j - p)
\]

(3)

where \( b \) and \( a \) are the background and star count rates (counts/sample) and \( p \) is the star position (transit time) in units of \( v \Delta t \) (\( \Delta t \)).

The counts \( \{n_j\} \) will be processed by feeding them through a linear filter \( F(x) \) and analysing the filtered sequence

\[
y_j = \sum_x F(x) n_{j+x}
\]

(4)

where \( x \) takes on integer values only. The following properties of the filter are particularly desirable:

(i) the expected filter output is zero when there is no star present (\( a = 0 \)), i.e. the output is insensitive to a uniform background (\( b \));

(ii) when a star is present at position \( p \), the output should have a peak at point \( p \) with expected amplitude \( E(y_p) = a \);

(iii) other positive local maxima (side lobes) produced by the star should be as much attenuated as possible compared to the main lobe; ideally \( E(y_j) \leq 0 \) for \( |j - p| > w \) = half width of main lobe.

We shall consider here a particular class of filters \( F(x) \) which can be realized as folded filters and therefore can be written in the form

\[
F(x) = \sum_{i=-\infty}^{\infty} G_i H(x - mi)
\]

(5)

analogous with (2). The numbers \( G_i \) are however not restricted to 0 or 1, but may be any real value. The elementary filter curve \( H(x) \) will typically resemble \( h(x) \), as required for the least-squares filter.
Inserting (3) and (5) in (4) we have

\[ E(y_j) = b \sum_i C_i \sum_x H(x) + a \sum_i \sum_k g_{i-k} A[j-p+m(i-k)] \]

with

\[ A(\xi) = \sum_x H(x) h(x + \xi) = H \ast h(\xi) \]

being the cross-correlation of \( H \) and \( h \). It is seen that the first requirement (i) is satisfied if

\[ \sum_i G_i = 0. \]

Eqn (6) then becomes

\[ E(y_j) = a \sum_i \sum_k g_{i-k} A[j-p+m(i-k)] \]

which is evidently the convolution of \( A \) and \( C \ast g \) with lag \( j-p \). At zero lag \( (j = p) \) we get

\[ E(y_p) = a \sum_i \sum_k g_{i-k} A[+m(i-k)] \]

and it is clear that the second requirement (ii) can be satisfied by suitable normalization of \( H(x) \).

If \( A(\xi) \) has a single maximum for \( \xi = 0 \) and \( A(0) \gg A(\pm m) \), then the relative strengths of the various maxima of \( E(y_j) \) will be determined by \( C \ast g \) (Fig. 1). Given the slit pattern \( g \), the third requirement (iii) then reduces to selecting the numbers \( G_i \) such that the cross-correlation sequence

\[ \Gamma_j = C \ast g(j) = \sum_i G_i g_{i-j} \]

has maximum side lobe suppression while subject to the constraint (8).

The following two constraints are furthermore introduced, although they are not strictly necessary:

\[ G_i = 1 \text{ if } g_i = 1 \]

\[ G_i < 0 \text{ if } g_i = 0 \]
Note that (12) implies $\Gamma_0 = N$ (number of slits).

The optimization of slit pattern and filter can now be divided in two parts:

I. Filter optimization

Given the slit pattern $S = \{g_i\}$, find the filter coefficients $\{G_i\}$ for which

$$\varepsilon(S) = \max \{\Gamma_j, j \neq 0\}$$

is minimized, subject to the constraints (8), (12), and (13).

II. Pattern optimization

Given $N =$ number of slits and $M =$ maximum pattern length (i.e. distance between extreme slits $\leq M$ samples), find the pattern with minimum $\varepsilon(S)$. The restriction on pattern length can be written

$$S \leq S_{\text{max}} = 2^{M+1} - 1,$$

so that this part simply consists in optimizing the filter coefficients successively for $S = 1, 3, 5, \ldots, S_{\text{max}}$ (skipping $S$ for which the number of slits is $\neq N$ or $R(S) < S$).

2. FILTER OPTIMIZATION

Rather than writing down the rather awkward general formulae, we shall use the specific pattern in Fig. 1 ($S = 147$) as an example. According to (12) we have

$$G_0 = G_1 = G_4 = G_7 = 1.$$  \hspace{1cm} (16)

Since the length of the pattern is

$$L = \text{entier}(2 \log S) = 7$$  \hspace{1cm} (17)

there is clearly no point in having $G_i \neq 0$ for $i < -L$ or $i > 2L$. Thus only $G_i$ for $i = -7, -6, -5, -4, -3, -2, -1, 2, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14$ need to be determined. The sum in (11) is effectively taken over $i = 0, 1, 4,$ and 7; e.g. for $j = -3,$
\[ \Gamma_{-3} = G_{-3} + G_{-2} + G_{1} + G_{4} = G_{-3} + G_{-2} + 2 \]  

(18)

using (16). Eqn (14) now implies

\[ \Gamma_{-3} \leq \epsilon \]  

(19)

or

\[-G_{-3} - G_{-2} + \epsilon \geq 2 \quad (j = -3).\]  

(20)

From the different conditions \( \Gamma_{j} \leq \epsilon \), i.e. for \( 0 < |j| \leq L \), we obtain the system of inequalities

\[
\begin{bmatrix}
-1 & -1 & 0 & 0 & -1 & 0 & 0 & . & . & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & -1 & -1 & 0 & 0 & -1 & 0 & . & . & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & -1 & 0 & 0 & . & . & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & -1 & 0 & 0 & . & . & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & . & . & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & . & . & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & . & . & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
G_{-7} \\
G_{-6} \\
G_{-5} \\
G_{-4} \\
G_{-3} \\
G_{-2} \\
G_{-1} \\
G_{2} \\
G_{3} \\
G_{4} \\
G_{5} \\
G_{6} \\
G_{7} \\
G_{8} \\
G_{9} \\
G_{10} \\
G_{11} \\
G_{12} \\
G_{13} \\
\epsilon
\end{bmatrix}
\geq \begin{bmatrix}
1 \\
1 \\
0 \\
1 \\
2 \\
0 \\
1 \\
0 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1
\end{bmatrix}
\]  

(21)

where certain columns and rows have been left empty to emphasize the structure of the system.
The optimum filter is obtained by minimizing \( \varepsilon \) subject to the constraints (21) and (22) - (24):

\[
- \Sigma_i G_i = N = 4 \quad \left\{ \begin{array}{l}
-7 \leq i \leq 14, \ i \neq 0, 1, 4, 7 \\
G_i \leq 0 \\
\varepsilon \geq 0
\end{array} \right. \quad (22)
\]

The system (21) can be somewhat simplified. For instance the third inequality \( (j = -5) \),

\[
- G_{-5} - G_{-4} - G_{-1} - G_{-2} + \varepsilon \geq 0 \quad (21.3)
\]

is obviously satisfied if (23) and (24) are, and can be deleted; similarly for the other rows with right-hand side = 0 \( (j = -2, 2, 5) \). After this it will be found that \( G_9 \) and \( G_{12} \) no longer occur in (21), so the number of unknowns can be reduced to 17. Introducing the new variables

\[
\begin{align*}
X_1 & = -G_{-7} \\
X_2 & = -G_{-6} \\
X_3 & = -G_{-5} \\
X_4 & = -G_{-4} \\
X_5 & = -G_{-3} \\
X_6 & = -G_{-2} \\
X_7 & = -G_{-1} \\
X_8 & = -G_{-2} \\
X_9 & = -G_3 \\
X_{10} & = -G_5 \\
X_{11} & = -G_6 \\
X_{12} & = -G_8 \\
X_{13} & = -G_{10} \\
X_{14} & = -G_{11} \\
X_{15} & = -G_{13} \\
X_{16} & = -G_{14} \\
X_{17} & = \varepsilon
\end{align*}
\]

The problem can be formulated as follows:

Minimize

\[
Z = Wx_1 + Wx_2 + \ldots + Wx_{16} + x_{17}
\]

subject to the constraints

\[
\begin{bmatrix}
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10} \\
x_{11} \\
x_{12} \\
x_{13} \\
x_{14} \\
x_{15} \\
x_{16} \\
x_{17}
\end{bmatrix}
\geq
\begin{bmatrix}
1 \\
1 \\
1 \\
1 \\
1 \\
2 \\
1 \\
1 \\
1 \\
1 \\
2 \\
1 \\
1 \\
1 \\
1 \\
1 \\
4
\end{bmatrix}
\quad , \quad x_i \geq 0
\]

Here $W$ is a large positive number which allows the equality constraint (22) to be written as the inequality in the last row of (27),

$$x_1 + x_2 + \ldots + x_{16} \geq 4.$$  \hspace{1cm} (27.11)

Since there will be a heavy penalty ($W$) in $Z$ if $s = x_1 + x_2 + \ldots + x_{16} - 4 > 0$, the minimization of $Z$ will automatically give a solution with $s = 0$ provided $W$ is large enough. ($W = 10$ was used in the computations reported below.)

Equations (26) - (27) are in the standard form of a minimization problem in linear programming, and the simplex algorithm is immediately applicable for its solution. For the pattern in our example the solution is

$$x = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
2/3 \\
1 \\
0 \\
0 \\
1 \\
0 \\
2/3 \\
2/3 \\
0 \\
0 \\
1/3
\end{bmatrix}$$  \hspace{1cm} (28)

yielding the sequences $\{G_1\}$ and $\{\Gamma_1\}$ shown in Fig. 1. Thus $\epsilon(147) = \frac{1}{3}$ and the side lobe suppression factor is $N/\epsilon = 12$. This is however far from the best pattern with $N = 4$ slits, cf. Table 1.

3. RESULTS

Table 1 give some data on slit patterns and filters obtained by systematic search of patterns with $N = 4$ slits ($S \leq 1023$), $N = 5$ or 6 slits ($S \leq 8191$), and $N = 7$, 8, or 9 slits ($S \leq 22227$). The corresponding sequences $\{g_1\}$, $\{G_1\}$, and $\{\Gamma_1\}$ are shown in Fig. 2.

The search was limited only by the available computer time (about 15 hours on the mini-computer HP21MX). Considering that very good patterns are ob-
Table 1. Data for some slit patterns and filters with efficient side lobe suppression. Explanations:

N  = number of slits
L  = pattern length, distance between extreme slits in units of \(mv\Delta t\)
S  = signature of pattern
\(\varepsilon\)  = maximum side lobe amplitude (main lobe \(= N\) units)
N/\(\varepsilon\)  = side lobe suppression factor
K1  = filter length, distance between extreme non-zero filter points in units of the basic mesh distance \(mv\Delta t\)
K2  = number of negative filter points (\(G_i < 0\))
S_{max}  = pattern search complete for \(S \leq S_{max}\)
\(Q = \frac{[\sum_i G_i^2]}{[\sum_i G_i^2 g_i]}\)  = filter's sensitivity to background level

<table>
<thead>
<tr>
<th>N</th>
<th>L</th>
<th>S</th>
<th>(\varepsilon)</th>
<th>N/(\varepsilon)</th>
<th>K1</th>
<th>K2</th>
<th>S_{max}</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>(\infty)</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>2.00000</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>(\infty)</td>
<td>3</td>
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<td>-</td>
<td>2.00000</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>11</td>
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<td>(\infty)</td>
<td>6</td>
<td>3</td>
<td>-</td>
<td>2.00000</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>83</td>
<td>0.1111</td>
<td>36.00</td>
<td>15</td>
<td>9</td>
<td>1023</td>
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</tr>
<tr>
<td>5</td>
<td>11</td>
<td>2437</td>
<td>0.1429</td>
<td>35.00</td>
<td>28</td>
<td>15</td>
<td>8191</td>
<td>1.43265</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>2215</td>
<td>0.1750</td>
<td>34.29</td>
<td>26</td>
<td>14</td>
<td>8191</td>
<td>1.63306</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>18599</td>
<td>0.1772</td>
<td>39.51</td>
<td>28</td>
<td>19</td>
<td>22227</td>
<td>1.54147</td>
</tr>
<tr>
<td>8</td>
<td>11</td>
<td>2479</td>
<td>0.2609</td>
<td>30.67</td>
<td>19</td>
<td>9</td>
<td>22227</td>
<td>2.11531</td>
</tr>
<tr>
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<td>22211</td>
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<td>35.48</td>
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</tr>
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<td>20061</td>
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<td>26</td>
<td>14</td>
<td>22227</td>
<td>1.92497</td>
</tr>
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</table>

tained for \( L \approx 2N \) (at least for \( N = 5 \) and \( 6 \)), it is very likely that even more efficient patterns can be found for \( N = 8 \) or \( 9 \) if the search is extended to longer patterns.

4. PHOTOMETRIC AND POSITIONAL ACCURACY

The estimated amplitude (\( \hat{a} \)) and position (\( \hat{p} \)) are subject to random errors in particular from the photon noise. The expected mean errors \( \sigma_{\hat{a}}, \sigma_{\hat{p}} \) can be analytically evaluated by using the following assumptions.

(A) The single slit response function \( h(x) \) is represented by a gaussian of height \( u \leq 1 \) and standard deviation \( z \):

\[
h(x) = u \exp(-x^2/2z^2).
\]

(B) The elementary filter curve \( H(x) \) is taken to be a gaussian with the same standard width,

\[
H(x) = U \exp(-x^2/2z^2),
\]

but with a height \( U \) determined from the normalization constraint

\[
\mathbb{E}(y_p) = a \quad [\text{cf. (10)}].
\]

(C) The basic mesh distance \( m\Delta t \) is large enough compared with \( z \) so that \( A(l) \approx 0 \) for \( |l| \geq m \).

(D) The sampling distance \( v\Delta t \) is on the other hand small enough compared to \( z \) that various sums can be represented by integrals.

Under these assumptions the positional mean error \( \sigma_{\hat{p}} \) and the photometric mean error in magnitudes, \( \sigma_m = (2.51 \log e) \sigma_{\hat{a}} / \hat{a} \), are (Lindgren, 1982 Feb 27):

\[
\sigma_m^2 = \frac{(2.51 \log e)^2}{\sqrt{\pi} N a u z \left[ (2/3)^{1/2} + Q \frac{b}{au} \right]} 
\]

\[
\sigma_p^2 = \frac{2 z^2}{\sqrt{\pi} N a u z \left[ (2/3)^{3/2} + Q \frac{b}{au} \right]} 
\]

where

\[
Q = \frac{\Sigma_i G_i^2}{\Sigma_i G_i^2 g_i} 
\]

(33)
indicates the sensitivity of the filter to background counts and is given in Table 1. In (31)-(32) we have also used that \( \frac{\Sigma_i g_i g_i}{\Sigma_i G_i G_i} = N \).

The Q-factor may be a very important consideration when comparing the different slit patterns and filters. For the faintest stars detectable, \( b/(au) > 1 \) so that the second term in brackets in (31) - (32) is the dominating one. If the background level is proportional to the number of slits,

\[ b = cN, \tag{33} \]

we find that, in the faint limit, \( \sigma_p \) and \( \sigma_m \) are proportional to \( \sqrt{Q} \) but independent of \( N \). The relatively large \( Q = 2.11531 \) for the 8-slit pattern \( S = 2479 \) seems to be a serious drawback of this otherwise attractive pattern (it is short and the filter relatively simple for so many slits). To investigate this point further, the mean errors \( \sigma_p \) and \( \sigma_m \) have been computed for the magnitude interval \( B = 6 \) to 12 and for the patterns \( S = 83 \) (4 slits), \( S = 18599 \) (7 slits), and \( S = 2479 \) (8 slits), using

\[ b = 0.25 N \text{ counts/sample (}\Delta t = 1/600 \text{ s)} \]

\[ a = 4 \times 10^{-0.6(B-10)} \]

\[ u = 0.6 \]

\[ z = 1.5 \text{ samples = 0.375 arcsec}, \]

cf. Lindegren, 1981 June 10. The results (Fig. 3) show remarkably small differences between the three patterns; the 7-slit pattern is however always best, but the 4-slit pattern is almost as good for faint stars, while the 8-slit pattern is almost as good for bright stars. The 4 slits are better than the 8 slits for \( B > 10 - 10.5 \). The break-even point is shifted to even brighter stars for enhanced background; e.g. with 10 times more background light the 4-slit pattern would be preferable for \( B > 7.5 \) to 8! The 4 slits have a number of further advantages over the 8 slits, which should render this pattern a serious candidate: the shorter pattern is more tolerant to deviations from the nominal spin rate; the filter is extremely simple and can be very efficiently implemented; the side-lobe suppression factor is high (36) and the negative side lobes moderately strong; finally, thanks to the shorter pattern a somewhat longer mesh distance \( (m) \) can be realized with less overlap of adjacent slit response functions, which should improve the accuracy especially for inclined slits.
5. ON THE ARRANGEMENT OF VERTICAL AND INCLINED SLITS

In the earlier study of non-periodic slit patterns (Lindegren, 1981 June 10) I suggested that "orthogonal" patterns be used for the two slit groups of each star mapper, so that feeding the signal through two different filters would decide whether a transit was at one group or the other (vertical or inclined slits). Since then I have been convinced that such patterns tend to be unnecessarily long and complex and the filters rather bad (e.g. in terms of side-lobe suppression). Moreover, this approach is computationally inefficient in that two filters are needed, each twice as long as for a single pattern.

Using coincidence detection seems to be much more efficient, especially since this will also remove most "false detections" in the TYCHO mode. Coincidence detection requires two parallel slit groups, so that the time difference between transits can be accurately predicted from the gyro rates (without using other attitude information). The parallel groups can either be between the two star mappers or within each star mapper. The first alternative is not so good because of the longer time difference and the different colour responses of the two star mappers. My proposal here is to use for each star mapper one long vertical group of four slits, and two shorter, parallel, inclined groups, each with the same pattern of four slits (Fig. 4e - f). This will at the same time improve the accuracy in the determination of the vertical coordinate (ζ), since there are effectively 8 inclined slits, while the total slit area is only moderately increased. The separation between each slit group must be at least 13m (cf. Fig. 2a), so that the positive side lobes of adjacent transits do not overlap.

Even if coincidence detection is primarily intended within each star mapper, one should still arrange the two star mappers in such a way that coincidence detection between them remains a possibility.
FIGURE 2a. Slit pattern ($G_i$), filter coefficients ($g_i$) and their convolution ($\Gamma_i$).
FIGURE 2b.

2437
FIGURE 3a. Photometric accuracy versus B-magnitude for three slit patterns.
FIGURE 3b. Positional accuracy versus B-magnitude for three slit patterns.
FIGURE 4a.
Original (Phase A) version.

FIGURE 4b.
Modified for TYCHO.

FIGURE 4c.
Improved version of Fig. 4b. This allows coincidence detection on inclined slits between star mappers.

FIGURE 4d.
Unbroken slits would greatly simplify TYCHO reductions.

FIGURE 4e.
Proposed arrangement: one vertical group and two parallel inclined groups. The angle $\theta$ is made as large as possible (up to 45°).

FIGURE 4f.
The same pattern of four slits is used for all three groups. Coincidence detection is used to discriminate between vertical and inclined transits.
FIGURE 5a. Realization of $S = 83$ filter ($N = 4$, $N/\varepsilon = 36.0$). 22 additions and 7 multiplications per sample.
FIGURE 5b. Approximative realization of $S = 18599$ filter ($N/c = 37^{1/3}$). 34 additions and 17 multiplications per sample.
FIGURE 5c. Realization of $S = 2479$ filter ($N = 8$, $N/\varepsilon = 30^{2/3}$). 26 additions and 13 multiplications per sample.