HIPPARCOS

Abscissa errors caused by periodic variations of the basic angle

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Summary

Variations of the basic angle with a fundamental period close to the spin period of a few hours will cause systematic abscissa errors along the great circle scans. The sensitivity of the abscissa solutions to such variations depends very much on the actual value of the basic angle. This effect should be considered when the basic angle is selected.

The direction towards the sun, as seen from the satellite, is specified by the two angles (Fig. 1)

\[ \xi = \text{revolving scanning angle} \]
\[ v = \text{solar azimuth angle}. \]

According to the revolving scanning law, \( \xi \) is fixed while \( v \) varies almost uniformly with time, going through 360° in a few hours. Because of the thermal impact of the solar radiation one must expect the basic angle to vary, at some level, periodically with \( v \). The effect of such a variation on the derived star abscissae along the scanning great circle was dealt with in an earlier note (1977 Dec 4). Since the results are still valid and have some bearing on the choice of basic angle, I give below an updated version of that note.

If the basic angle \( \gamma \) varies periodically with \( v \) around a mean value \( \gamma_0 \), we have

\[ \gamma(v) = \gamma_0 + \sum_{k=1}^{\infty} [a_k \cos kv + b_k \sin kv]. \]  

Now let \( w_i \) be the true abscissa of a star, measured with origin at the point on the scanning great circle nearest to the sun (Fig. 1), and \( w_i + \Delta w_i \) the abscissa derived from observations with a varying basic angle. The relevant deterministic part of the observation equation for a connexion of two stars belonging to opposite fields (p and f) is

\[ \Delta w_f - \Delta w_p = \sum_{k=1}^{\infty} [a_k \cos kv + b_k \sin kv], \]

where \( v = \frac{1}{2}(w_f + w_p) \). Putting

\[ \Delta w_i = \sum_{k=1}^{\infty} [A_k \cos kw_i + B_k \sin kw_i], \]  

and using the approximate abscissae \( w_p = v + \frac{1}{2} \gamma, w_f = v - \frac{1}{2} \gamma \), we get the solution

\[ A_k = \frac{1}{2}b_k / \sin(k\gamma/2), \quad B_k = -\frac{1}{2}a_k / \sin(k\gamma/2). \]

It can be shown that inclusion of \( p-p \) and \( f-f \) connexions does not alter \( A_k, B_k \) very much, except by removing the singularities at \( k\gamma/2 = n\pi \) (\( n = \) integer).
If a single harmonic \((k)\) is considered, we have the following relation between the amplitude of the abscissa errors, \(\Delta a = \sqrt{(a_k + b_k)}\), and the corresponding amplitude of the basic angle variation, \(\Delta \gamma = \sqrt{(a_k + b_k)}\):

\[
\frac{\Delta a}{\Delta \gamma} = \frac{1}{2|\sin(k\gamma/2)|}.
\]

(5)

The common logarithm of this ratio has been plotted versus \(\gamma\) in Fig. 3, for \(k = 1, 2, \ldots, 6\).

It is interesting to compare Fig. 3 with a plot of the relative variance of the abscissae as function of \(\gamma\), computed of course under the assumption that \(\gamma\) is constant (Fig. 4). Both figures show that simple rational angles (60°, 72°, 90°, etc.) should be avoided; however, the peaks in Fig. 3 are much wider, and the regions to be avoided depend of course on which harmonics are expected and their relative amplitudes.

Although the abscissa errors represented by the low-order coefficients \(A_k, B_k\) are of a systematic nature, it is not obvious that they will appear in the final astrometric data as systematic errors, e.g. a distortion of the positional system. This is related to the important question whether these coefficients can be separated from astrometric data and hence solved for in 'Step 3' of the scientific data processing. With the exception of \(B_1\) this is perhaps possible, as explained below, provided \(A_k, B_k\) are reasonably constant throughout the mission.

It is easily seen that the parallactic displacement of a star with parallax \(\pi_i\) is

\[
\Delta \omega = -\pi_i \sin \xi \sin \omega_i.
\]

(6)

Therefore, if all scans across this star are distorted by the same coefficient \(B_1\), the solution for its parallax will be biased by \(\Delta \pi = -B_1 / \sin \xi\). The same bias will be obtained for all stars. Such a global zero-point error of the parallaxes cannot be internally detected by the scientific data processing, since all measurements are perfectly consistent with the reduction model. Only by introducing external information, e.g. photometric distances to some stars, will it be possible to fix the zero point.

For the other coefficients \((A_k, k > 0, \text{and } B_k, k > 1)\) there is no obvious correlation with astrometric data (this remains to be shown). If so, they may be determined by comparing scans along the same great circle but with the sun in different positions (Fig. 2). For a given star the abscissa errors will be the same, but with opposite signs, in two scans for which the solar azimuth difference is an odd multiple of \(\pi/k\) radians. The same effect occurs when the sun is in opposite positions. With the revolving scanning law many such pairs of scans are likely to occur, although there will sometimes be a time difference of six months or more between them.

Even if the variations of the basic angle are expected to be negligible, such tests for systematic effects must of course be included in the scientific data processing.
Fig. 1. Sun-spacecraft geometry: $z =$ nominal spin axis; $p, f =$ preceding, following viewing direction; $x =$ midpoint between $p$ and $f$; $\xi =$ revolving scanning angle; $\nu =$ solar azimuth angle; $w =$ (sun-related) abscissa of a star in the preceding field.

Fig. 2. If approximately the same great circle is scanned with the sun at two different positions, the abscissa errors will be phase shifted by $\Delta \nu$ between the two scans. If $\Delta \nu \approx \frac{n\pi}{k}$, where $n$ is odd, the geometry is ideal for determination of the $k$:th harmonic of the abscissa errors.
Sensitivity of abscissa solution to periodic variations of basic angle

$\Delta \gamma$ = amplitude of basic angle variation (one harmonic)
$\Delta \alpha$ = amplitude of resulting periodic abscissa error
k = order of harmonic (fluctuation period = $2\pi/k$)
RELATIVE VARIANCE OF ABSCISSA SOLUTION VERSUS BASIC ANGLE (GAMMA)

N = 780 STARS PER SCAN
M = 4 STARS PER FOV

(Lindegren, 1981 May 1)

FIG. 4