DERIVATION OF POSITIONS AND PARALLAXES OF STARS FROM SIMULATED OBSERVATIONS WITH A SCANNING ASTROMETRY SATELLITE

4. Comparison with theoretical predictions

The average mean error of e.g. the parallaxes determined in a simulation experiment is directly proportional to the assumed dispersion of individual 'observations' (i.e. the simulated measurements of the coordinates of star images in a frame). If the corresponding least-squares problem is well conditioned, it should be possible to estimate the constant of proportionality by means of relatively simple geometrical and statistical considerations. These must involve two kinds of assumptions. Firstly, the strict observation equations must be reformulated and simplified to the point where only terms which are absolutely essential to the principle of observation are retained. The assumption is that other terms, being of a second-order nature, contribute little to the variance of the results, although the results themselves may depend critically on the inclusion of such terms. Secondly, it must be assumed that sensible estimates can be obtained if certain quantities in the simplified normal equations (e.g. the actual number of observations of a particular star) are replaced by more easily computable averages (e.g. the average number of observations per star). Moreover, probabilistic arguments must sometimes be invoked for the estimation of such averages: quantities are formally treated as random variables, their probability density functions are computed, and the expectation adopted as mean value.

Since theoretical estimates of this kind are extremely convenient for optimisation purposes and in addition form the basis for extrapolating the results of small-scale simulation experiments to the full-scale problem, it is necessary to test the validity of the various assumptions involved by comparing the results of theoretical predictions with simulation results derived from a much more complete model of observations and reductions. This is indeed one of the main reasons for undertaking the presently reported simulation experiments.
The numerical experiments described above apply a least-squares solution in a very straight-forward way: the 'observations' \((n, \zeta)\) are introduced directly into the observation equations for the astrometric parameters (positions and parallaxes). For the present discussion it is however convenient to split the problem into two quite distinct parts. Firstly, we consider the combination of observations obtained during the full revolution of the instrument around its spin axis \((z)\), which constitute a scan of a small fraction of the stars along a great circle on the sky. By least squares, we solve for one positional component of the stars, viz. along the scanning great circle. Such an intermediary solution is motivated by (a) that the stars may be considered stationary during such a short period of time, and (b) that the method of observation provides information almost exclusively in the direction parallel to the scan, and very little in the perpendicular direction. In the second part of the solution, the results from the first part are treated as the input ('observations') for a second least-squares solution in which the astrometric parameters are the unknowns. This is now a trivial problem, since it is possible to treat each star separately.

Consider first the solution along a great circle scan. Let \(N\) be the total number of stars to be observed by the satellite. These are assumed to be (in a statistical sense) uniformly distributed over the entire celestial sphere. Thus, if \(\Theta\) is the solid angle of each of the two superposed fields of view, the average number of star images in a frame will be

\[
m = \frac{N\Theta^2}{2\pi}.
\]  

(4.1)

As the instrument rotates one turn around the \(z\) axis (which for the moment can be assumed to be at a fixed direction), the two fields of view scan a strip of solid angle \(2\pi\Theta\) along a great circle. The corresponding set of observations is called a scan. Clearly, the number of stars encountered in a scan is, on the average,

\[
n = \frac{N\Theta}{2}.
\]  

(4.2)

Note that the average number of star images observed in a scan is \(2n\), since each star is observed twice (preceding and following fields).
An observation is a measurement of the coordinates \((n, \zeta)\) of a star image in the focal plane of the telescope, referred to a given instant of time. A frame is a set of observations referred to the same instant. We shall assume that all stars visible in the combined field of view at a given instant are observed, so that the average number of observations per frame is \(m\). Let \(M\) be the number of frames per scan. A certain overlap of successive frames is required to secure the connexion of different parts of the scan. If \(Q_f\) is the degree of overlap \((0 < Q_f < 1)\), we have

\[
M = \frac{2\pi}{(1 - Q_f)\phi}.
\]  

Let \(\sigma_n, \sigma_\zeta\) denote the average mean errors of the two components of an observation. It is a basic feature of the type of instrument considered that information is gained mainly in the direction of scanning, i.e. that \(\sigma_n \ll \sigma_\zeta\). In fact, the observations normal to the scan \((\zeta)\) are used mainly to derive the instantaneous pole of scanning \((\lambda_2, \beta_2)\), which in turn enters the observation equations in some correction terms of a second-order nature. Consider the case where two stars in a frame belong to opposite fields \((p\) and \(f)\). Inspection of the detailed observation equations for \(n_p, \zeta_p, n_f,\) and \(\zeta_f\) shows that the errors in \(\zeta\) will be transferred to \(n\) according to

\[
\Delta n_p = -\tan \zeta_p \frac{\Delta \zeta_p \cos \gamma - \Delta \zeta_f}{\sin \gamma},
\]

\[
\Delta n_f = -\tan \zeta_f \frac{\Delta \zeta_p - \Delta \zeta_f \cos \gamma}{\sin \gamma},
\]

where \(\gamma\) is the basic angle. To account for \(\sigma_\zeta\) we may consider that the observed quantity is \(n - \Delta n\), having variance

\[
\sigma_n^2 = \sigma_n^2 + \frac{2}{m} \left< \tan^2 \zeta \right> \frac{1 + \cos^2 \gamma}{\sin^2 \gamma} \sigma_\zeta^2 = \sigma_n^2 + \frac{\phi^2}{6m} \frac{1 + \cos^2 \gamma}{\sin^2 \gamma} \sigma_\zeta^2,
\]

since \(m/2\) independent pairs of stars may be used to determine \((\lambda_2, \beta_2)\), and \(\zeta\) is uniformly distributed over the field width \(\pm \phi\).

However, since the pole of scanning may change progressively during the scan, and is in any case not known with great precision, we shall prefer to introduce a fixed and exactly defined reference great circle for the observations of a scan. The pole of this circle \((\lambda_p, \beta_p)\) is chosen to be near the mean pole of scanning during this particular scan; thus, the scanning direction is always nearly parallel to this circle, and the mean error of the observation component along the reference circle is practically equal to \(\sigma_n\). The introduction of a reference great circle is equivalent to transforming star coordinates into a provisional, but completely well-defined, reference coordinate system, in which the components of position may be called abscissa (the angle along the reference great circle) and ordinate (the perpendicular coordinate).
After combining the observations of a scan into a least-squares solution for the abscissae of stars \( \alpha_i \), it is found that the average abscissa mean error \( \sigma_\alpha \) is related to the average observational mean error \( \sigma_n \) by an equation

\[
\sigma_\alpha^2 = V \frac{n}{nM} \sigma_n^2 = \frac{1}{2}(1 - Q_f) V \sigma_n^2,
\]

(4.6)

in which the factor \( V \) is of the order of, but slightly greater than, unity. Its precise value depends on the numbers \( n, m, \) and \( M \), as well as on the detailed structure of the normal equations. The factor \( V \) is estimated here by means of an idealised system of observation equations which may be solved by a Fourier method similar to the one used for determination of the division errors on a graduated circle (e.g. Høg, 1960).

Let the stars of a scan be numbered \( i = 0, 1, \ldots, n-1 \) according to increasing abscissa, and let us assume that the stars are sufficiently regularly spaced along the reference great circle that the star images successively passing the centre of the combined field of view have star number \( \ldots, i, i+g, i+1, i+g+1, i+2, \ldots \), where \( g \) is an integer related to the basic angle \( \gamma \) of the telescope: \( \gamma = 2\pi(g - \frac{1}{2})/n \). (All indices are understood modulo \( n \).) Assume, furthermore, that the frame overlap is \( Q_f = (m-1)/m \), i.e. exactly one star image is shifted out with each new frame, and another image entered. The observation equations of two successive frames are then essentially of the form

\[
\begin{align*}
\alpha_i & - \omega_{2i} - \frac{1}{2} \gamma = \ldots \\
\alpha_{i+1} & - \omega_{2i+1} - \frac{1}{2} \gamma = \ldots \\
\vdots & \\
\alpha_{i+(m-1)/2} & - \omega_{2i} - \frac{1}{2} \gamma = \ldots (m \text{ odd}) \\
or & \alpha_{i+g+(m-2)/2} - \omega_{2i} + \frac{1}{2} \gamma = \ldots (m \text{ even}) \\
\end{align*}
\]

frame no. \( 2i \)

\[
\begin{align*}
\alpha_{i+g} & - \omega_{2i+1} + \frac{1}{2} \gamma = \ldots \\
\alpha_{i+1} & - \omega_{2i+1} - \frac{1}{2} \gamma = \ldots \\
\alpha_{i+g+1} & - \omega_{2i+1} + \frac{1}{2} \gamma = \ldots \\
\vdots & \\
\alpha_{i+g+(m-1)/2} & - \omega_{2i+1} + \frac{1}{2} \gamma = \ldots (m \text{ odd}) \\
or & \alpha_{i+m/2} - \omega_{2i+1} - \frac{1}{2} \gamma = \ldots (m \text{ even}) \\
\end{align*}
\]

frame no. \( 2i+1 \)
\( \omega_i \) is the abscissa of the midpoint between the centres of the two fields of view in the \( i \text{th} \) frame; \( \omega_i \pm \frac{1}{2} \gamma \) are thus the abscissae of the two field centres in the adopted approximation.

Forming the normal equations and eliminating \( \omega_i \) and \( \gamma \) we obtain the following reduced normal equation for the unknown \( \alpha_i \):

\[
2(m-1)\alpha_i - \sum_{k=1}^{m-1} \frac{m-k}{m} \left( \alpha_i - g-(k-1)/2 + \alpha_i - g+(k+1)/2 + \alpha_i + g-(k-1)/2 + \alpha_i + g+(k-1)/2 \right) \tag{4.8}
\]

\[
+ \alpha_i + g+(k-1)/2 \right) - \sum_{k=2}^{m-1} \frac{m-k}{m} \left( 2\alpha_i - k/2 + 2\alpha_i + k/2 \right) = R_i \quad (even)
\]

where \( R_i \) to denote the right-hand member. In matrix notation the reduced normal equations may be written \( \underline{N} \underline{A} = \underline{R} \), where \( \underline{N} \) is a cyclic symmetric matrix of order \( n \). If the solution of this system is formally written \( \underline{A} = \underline{N}^{-1} \underline{R} \), we have the well-known result

\[
\sigma_{\alpha}^2 = (\underline{N}^{-1})_{ii} \sigma_{\eta}^2 \tag{4.9}
\]

where \( (\underline{N}^{-1})_{ii} \) is one of the \( n \) identical diagonal elements of \( \underline{N}^{-1} \).

The matrix \( \underline{N} \) is however singular (the rank is in general \( n-1 \)), but we may find a generalised inverse corresponding to the solution \( \underline{A} \) constrained by the linear equation \( \Sigma_i \alpha_i = 0 \). This is obtained by putting (for \( n \) even)

\[
\alpha_i = \sum_{l=1}^{n/2} \left( \alpha_1 \cos(2\pi il/n) + b_1 \sin(2\pi il/n) \right), \quad \tag{4.10}
\]

\[
R_i = \sum_{l=1}^{n/2} \left( A_1 \cos(2\pi il/n) + B_1 \sin(2\pi il/n) \right). \quad \tag{4.11}
\]

Introducing (4.10) and (4.11) into (4.8), simplifying, and equating the left and right members of each harmonic term separately, we find that

\[
a_1 = A_1/K_1, \quad b_1 = B_1/K_1, \quad 1 = 1, 2, \ldots, n/2, \quad \tag{4.12}
\]

where
\[ K_1 = 2(m-1) - \sum_{k=1}^{m-1} \frac{m-k}{m} 4\cos(\pi kl/n)\cos(2\pi l(g-\frac{1}{2})/n) - \]
\[ - \sum_{k=2}^{m-1} \frac{m-k}{m} 4\cos(\pi kl/n). \]  
\hspace{1cm} (4.13)

But the Fourier coefficients of \( R_1 \) are
\[ A_1 = 2 \left( \frac{n}{n-1} \right) \sum_{j=0}^{n-1} R_j \cos(2\pi j l/n); \quad B_1 = 2 \left( \frac{n}{n-1} \right) \sum_{j=0}^{n-1} R_j \sin(2\pi j l/n); \]  
\hspace{1cm} (4.14)

hence we find
\[ a_1 = 2 \left( \frac{n}{n-1} \right) \sum_{j=0}^{n-1} R_j \sum_{i=1}^{n/2} \cos(2\pi l(i-j)/n)/K_1, \]  
\hspace{1cm} (4.15)

or
\[ \left( \frac{N^{-1}}{n} \right)_{ij} = 2 \frac{n}{n-1} \sum_{i=1}^{n/2} \cos(2\pi l(i-j)/n)/K_1. \]  
\hspace{1cm} (4.16)

Combining this with (4.4) and (4.7) and \( 1 - Q_f = 1/m \), we have finally
\[ V = \frac{4m}{n} \sum_{l=1}^{n/2} 1/K_1. \]  
\hspace{1cm} (4.17)

Notwithstanding that the basic angle does not in general correspond to an integer value \( g \), we obtain a reasonable estimate of \( V(n, m, \gamma) \) by putting \( 2\pi (g-\frac{1}{2})/n = \gamma \) in (4.11). Fig. 4.1 shows the function \( V(n, m, \gamma) \) for \( \gamma = 68.5^0 \). It is assumed that the figure is representative also for odd \( n \) and for frame overlap factors different from \((m-1)/m\).

The second part of the problem is the combination of the different abscissa measurements of a particular star yielding its astrometric parameters. The observation equations must be written in differential form. Let \( \Delta \alpha, \Delta \beta, \Delta \omega \) be the differential corrections to the three astrometric parameters of a certain star, and let \( \Delta \alpha \) be the 'observed' abscissa correction. The observation equation becomes
\[ \Delta \alpha \cos \beta \sin \theta + \Delta \beta \cos \theta + \Delta \omega \sin \xi \sin u = \Delta \alpha, \]  
\hspace{1cm} (4.18)

where \( \theta \) is the position angle of the +\( \alpha \) direction at the star, \( \xi \) is the
angle between the Sun and the pole of the reference circle (pole of
scanning \( \lambda_z, \beta_z \)), and \( u \) is the angle at \( z \) between the Sun and the star
(Fig. 4.2).

In order to solve the normal equations we adopt the statistical
approximations

\[
\Sigma \sin^2 \theta = N_0 \langle \sin^2 \theta \rangle = N_0 P_\lambda(\xi),
\]

\[
\Sigma \cos^2 \theta = N_0 \langle \cos^2 \theta \rangle = N_0 P_\beta(\xi) = N_0 (1 - P_\lambda(\xi)),
\]

\[
\Sigma (\sin \theta \cos u)^2 = N_0 \sin^2 \xi <\sin^2 u> = N_0 P_\beta(\xi),
\]

\[
\Sigma \sin \theta \cos \theta = \Sigma \sin \theta \sin u = \Sigma \cos \theta \sin u = 0,
\]

in which \( N_0 \) is the average number of scans in which a star is observed
and \( \langle \rangle \) denote averages with respect to the different stars and different
observations of them. The mean variances of the astrometric parameters
are then \( \sigma_\alpha^2 / N_0 P_\lambda(\xi) \). It remains to estimate \( N_0 \), \( \langle \sin^2 \theta \rangle \), and \( \langle \sin^2 u \rangle \).

The fraction of stars covered by a scan is \( n/N = 1/\phi \). Thus \( N_0 = \phi N_s \),
where \( N_s \) is the total number of scans. This can be related to the scanning
law and the size of the field of view by the requirement that successive
scans must overlap by a fraction \( Q_s > 0 \) of the width of the field of view
at the points where the overlap is minimum. If \( S \) is the total length (in
radians) of the path described by the pole of scanning, we have \( N_s =
S/\phi(1 - Q_s) \) and hence

\[
N_0 = S/2(1 - Q_s).
\]

For a revolving scanning law \( (K, \xi) \), lasting \( T \) years, we find

\[
S = 2\pi T (1 + (K^2 - 1/2) \sin^2 \xi),
\]

which is roughly independent of \( \xi \) for \( K = 270^0/\xi \) and \( 20^0 \leq \xi \leq 45^0 \) (cf.
Table 4.1).

To estimate the global average \( \langle \sin^2 \theta \rangle \) we consider first stars at a
fixed latitude \( \beta \). For revolving scanning we may put

\[
\sin \beta_z = \sin \xi \sin \phi
\]
and assume that $v$ changes uniformly with time. Thus we shall take a mean with respect to a uniform distribution of $v$. There are however two more things to be taken into account. Firstly, the reference great circle will go through latitude $\beta$ only if $|\beta_z| < \frac{1}{2}\pi - |\beta|$. This condition is always satisfied for $|\beta| < \frac{1}{2}\pi - \xi$ (since $|\beta_z| \leq \xi$), but for $|\beta| \geq \frac{1}{2}\pi - \xi$ it imposes the restriction $|\sin \beta| < \cos \beta / \sin \xi$. Secondly, the frequency of observation at latitude $\beta$ is proportional to $|\sec \theta|$, since $\frac{1}{2}\pi - |\theta|$ is the angle of intersection between the reference great circle and the latitude parallel. The average of $\sin^2 \theta$ for stars at latitude $\beta$ is then

$$<\sin^2 \theta>_\beta = \frac{\int_{-\nu_0}^{\nu_0} \sin^2 \theta (1 - \sin^2 \theta)^{-\frac{1}{2}} \, d\nu}{\int_{-\nu_0}^{\nu_0} (1 - \sin^2 \theta)^{-\frac{1}{2}} \, d\nu}, \quad (4.23)$$

where $\nu_0 = \arcsin \left( \min(1, \cos \beta / \sin \xi) \right)$. Neglecting the ordinate of the stars we have

$$\sin \theta = \sin \beta_z / \cos \beta = \sin \xi / \sin \nu / \cos \beta, \quad (4.24)$$

and hence

$$<\sin^2 \theta>_\beta = \begin{cases} \frac{1 - E(\sin^2 \xi / \cos^2 \beta)}{K(\sin^2 \xi / \cos^2 \beta)} & \text{for } |\beta| < \frac{1}{2}\pi - \xi, \\ \frac{\sin^2 \xi}{\cos^2 \beta} \left(1 - \frac{E(\cos^2 \theta / \sin^2 \xi)}{K(\cos^2 \theta / \sin^2 \xi)}\right) & \text{for } |\beta| \geq \frac{1}{2}\pi - \xi. \end{cases} \quad (4.25)$$

$K$ and $E$ are the complete elliptic integrals of the first and second kinds. The global mean, which is a function of $\xi$ only, must be computed by numerical quadrature:

$$P_\lambda(\xi) = \int_{0}^{\pi/2} <\sin^2 \theta>_\beta \cos \beta \, d\beta. \quad (4.26)$$

The functions $P_\lambda(\xi)$ are given in Table 4.1.

When an arbitrary star is considered and an arbitrary scanning law (with $\xi$ given however), it is realised that there is no preference for any particular angle $u$; hence $<\sin^2 u> = \frac{1}{2}$ and

$$P_\xi(\xi) = \frac{1}{2} \sin^2 \xi. \quad (4.27)$$
The complete formula for estimating $\sigma_\lambda^2$, $\sigma_\beta^2$, and $\sigma_\omega^2$, combining eqs. (4.5), (4.6), and (4.20), becomes

\[
\sigma_\omega^2 = (1 - Q_f)(1 - Q_s) \frac{V}{\text{SP}_a(\xi)} \left( \sigma_n^2 + \frac{\sigma^2_\omega}{6m} \frac{1 + \cos^2 \gamma}{\sin^2 \gamma} \right),
\]

(4.28)

where $V$ is taken from Fig. 4.1 and $P_a(\xi)$ from Table 4.1.

A comparison of predictions according to (4.28) with the numerical results of the simulation experiments are given in Table 4.2 and Fig. 4.3.

---

Table 4.1. The weights of abscissa measurements with respect to the three astrometric parameters and as functions of $\xi$, the angle of the Sun from the pole of scanning. The table also gives S/T, the length of the path of the z axis on the sky for revolving scanning with $K = 270^\circ/\xi$.

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$P_\lambda(\xi)$</th>
<th>$P_\beta(\xi)$</th>
<th>$P_\omega(\xi)$</th>
<th>S/T</th>
</tr>
</thead>
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<tr>
<td>0°</td>
<td>.000</td>
<td>1.000</td>
<td>.000</td>
<td>30.3 rad/yr</td>
</tr>
<tr>
<td>10</td>
<td>.047</td>
<td>.953</td>
<td>.015</td>
<td>30.1</td>
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<td>.058</td>
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<tr>
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<td>.744</td>
<td>.125</td>
<td>28.9</td>
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<td>.207</td>
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<td>.460</td>
<td>.375</td>
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<td>.442</td>
<td>23.3</td>
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<td>.608</td>
<td>.392</td>
<td>.485</td>
<td>21.4</td>
</tr>
<tr>
<td>90</td>
<td>.606</td>
<td>.394</td>
<td>.500</td>
<td>19.4</td>
</tr>
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</table>
Table 4.2. Comparison of theoretically predicted mean errors with results of numerical simulation experiments. Mean errors are expressed in marcsec (0.001")

<table>
<thead>
<tr>
<th>Experiment no.</th>
<th>N</th>
<th>φ</th>
<th>n</th>
<th>m</th>
<th>ξ</th>
<th>T</th>
<th>σ_η</th>
<th>σ_ζ</th>
<th>σ_λ</th>
<th>σ_β</th>
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<th>σ_λ</th>
<th>σ_β</th>
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<td>15</td>
<td>26</td>
<td>2</td>
<td>30</td>
<td>0.5</td>
<td>10</td>
<td>100</td>
<td>6.9</td>
<td>4.1</td>
<td>-</td>
<td>5.6</td>
<td>3.3</td>
<td>8.0</td>
</tr>
<tr>
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<td>200</td>
<td>15</td>
<td>26</td>
<td>2</td>
<td>40</td>
<td>0.5</td>
<td>10</td>
<td>100</td>
<td>5.2</td>
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<td>-</td>
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<td>3.6</td>
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</tr>
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<td>-</td>
<td>4.4</td>
<td>3.4</td>
<td>5.9</td>
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<tr>
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<td>26</td>
<td>4</td>
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<td>1.0</td>
<td>10</td>
<td>100</td>
<td>3.7</td>
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<td>13</td>
<td>46</td>
<td>27</td>
<td>66</td>
<td></td>
</tr>
</tbody>
</table>

Average ratio of experimental m.e. to theoretical m.e. (excluding 10a-c): 1.17 1.15 0.68
Fig. 4.3.
Result of simulation [in units]

Fig. 4.3