DETECTION AND MEASUREMENT OF DOUBLE STARS
WITH AN ASTROMETRY SATELLITE

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ABSTRACT

The proposed astrometry satellite HIPPARCOS, in its present design, will observe stars by scanning their images across a periodic grid in the focal plane of a telescope. This comparatively simple method should work well and rather efficiently when observing simple, isolated stars, but may give rise to difficulties with the 20% expected double stars in any observing program (magnitude difference < 5, separation 0".25 - 15"). Good ability to detect and properly measure such double stars can however be gained at the expense of more computing and a larger grid period, giving only a moderate 20% loss of accuracy for single-star observations. At least half of these double stars are already known and existing information about them should be employed, but some 3500 new double stars will be discovered and measured by the satellite through decomposition of detector signals and analysis of residuals. The position of the photocentre is obtained for double stars closer than 0".25; its orbital motion will have small if not negligible effects on observed proper motions and parallaxes.

1. INTRODUCTION

An interesting and possibly significant by-product of a space astrometry mission would be a systematic search for double and multiple stars, covering the whole sky uniformly and with well-defined detection criteria. In practice, however, the value of an astrometry satellite for double-star observation is perhaps less than superficially expected: after all, the telescope is rather small (25 cm diameter), with a resolution not better than a few tenths of an arcsec, and the detector and method of observation (as now being foreseen) are not very suitable for observation of more than one star at a time. Nevertheless, a substantial fraction of the observed stars, about 20%, will be visual double stars of a kind that might have a bad influence on the overall astrometric accuracy, should they not be recognized as double and treated accordingly. Thus, apart from its own value, double-star detection will be a rather essential ingredient of a successful astrometry mission. It is this aspect, primarily, that is considered in this paper.
It is not obviously clear why double stars should present any problem at all with an astrometry satellite. The scientific instrument of the satellite HIPPARCOS, being studied by the European Space Agency (ESA, 1978), consists of four essential parts: complex mirror, telescope, grid, and detector. Using a modulating grid in the focal plane of the telescope is of course a compromise between science and feasibility. Ideally, one should have in the focal plane a two-dimen-
sional photon-counting detector with a resolution and positional stability and linearity corresponding to astrometric requirements for a field size of about one degree, i.e. of the order of $10^{-5}$ to $10^{-6}$. This would allow full employment of the astrometric information carried by each detected photon, leading to higher accuracy for individual stars and permitting simultaneous observation of many stars without mutual interference. Resolved double and multiple stars would cause no extra trouble; systems unresolved by the telescope must in any case be observed by their photocentres.

Unfortunately, such a device does not yet seem to exist. The proposed solution is to have a periodic grid maintaining the desired positional accuracy, and behind it a detector with almost completely relaxed accuracy requirements. When, due to the spinning motion of the satellite, the stellar images move across the grid, one samples in effect one or a few Fourier components of the focal-plane image in certain directions. But each Fourier component can give the intensity and position of one star only (through its amplitude and phase). If there is indeed only one star in the field of view the method works almost as efficiently as that with the ideal spatially resolving detector. But it is almost useless if there are more stars of signifi-
cant brightness in the field of view than there are Fourier components in the detector signal.

Therefore it is necessary to eliminate as many parasitic stars as possible from the field of view. This is done by the image dissector having a sensitive cathod spot corresponding to an instantaneous field of view (IFOV) as small as practicable, about 30" diameter. As shown by Figure 1, this eliminates quite efficiently undesirable field stars, but has practically no effect on double and multiple stars since their components are most likely separated by much less than 15".

Thus it should be expected that parasitic stars are present in about 20% of the observations, and then we have in general a double star. The probability of getting two or more visible physical compan-
ions in the IFOV is only 2%.

2. EFFECTS OF UNDETECTED COMPANIONS

A scan across a double star (not recognized as such) gives a one-dimen-
sional position of the system in the scanning direction. A two-dimen-
sional position is eventually obtained by combining several scans in different directions (Fig. 2). Since the image is not radially sym-
metric, the different scans (if more than two) generally do not define a unique position, even in absence of noise, but give rise to deter-
ministic residuals, depicted by the dispersion radius in Figure 2, as well as a certain bias (offset from the desired position of the primary). These will depend on the separation ($\rho$) and magnitude difference ($\Delta m$) between the components, as indicated in Figure 3. The effects of the undetected secondary are clearly different for unresolved ($\rho \leq 0^\prime25$) and resolved ($\rho \geq 0^\prime25$) double stars: for unresolved pairs the dispersion is zero and the bias proportional to the separation; in fact, the positions
Figure 1. Probability of getting a parasitic star within the instantaneous field of view (IFOV) as function of IFOV radius \( r \). Physical companions to the program star are considered separately from field stars (optical binaries) to emphasize their very different distributions of separation. A parasitic star is defined as having magnitude difference \( \Delta m < 5 \) relative the program star and separation \( 0''25 < \rho < 15'' \) from it. The distribution of binary separations is according to Heintz (1969) with median separation 0''1, that of field stars is for twice the average density of \( m_{pg} < 14 \) stars (Allen, 1973).

Figure 2. Observation of a star having an undetected secondary. Each scan across the system measures its coordinate along this scan and hence determines a line of position perpendicular to the scan. For a radially symmetric image, e.g. a single star, several such scans will intersect in a single point at the center of the image, disregarding measurement noise. But for a double star the intersection points define a small area (dashed circle) characterized by its expected offset from the desired position ("bias"), and an rms residual due to model mismatch ("dispersion radius").
Figure 3. Bias and dispersion radius (defined in Fig. 2) as functions of separation and magnitude difference. It is assumed that the double star is scanned many times in many different directions.

obtained are those of the photocentres. In contrast, for resolved pairs, the photocentres have no obvious meaning. The bias then oscillates between ± 0".2 dex (−0.4 Δm) and the dispersion radius is almost independent of separation 0".1 dex (−0.4 Δm). Thus, aiming at a precision of 0".002 in the final results, we should regard a companion with Δm < 5 and separation in the range 0".25 < ρ < 15" as a potentially disturbing parasite.

The photocentre observed for close binaries does not, in general, coincide with the centre of mass of the system, which should ideally be observed to yield accurate proper motions and parallaxes. The residual orbital motion of the photocentre may have some effects on measured proper motions and parallaxes because of the short interval of observation (a few years). Assuming circular, randomly oriented orbits with two main-sequence stars, and making use of the known distributions of orbital periods (Heintz, 1969) and absolute magnitudes (Allen, 1973), it is found that 10% of all 9th stars are really unresolved binaries for which the proper motion errors due to this effect are greater than 0".002/yr. The corresponding fraction for errors in excess of 0".01/yr is 0.2%. The effect is thus seen to be statistically small, but may lead to the discovery of a few hundred new astrometric binaries. The effect on measured parallaxes is noticeable only for resonant periods; the relative parallax error may exceed 10% for the 1% stars which are double with periods between 0.9 and 1.25 yr.

3. DETECTION OF DOUBLE STARS

One can envisage at least three largely independent ways to detect parasitic stars, working on different levels:

(1) **a priori** detection, i.e. known double stars (to be considered already when setting up the observing program);

(2) decomposition of the detector signal into the components of the individual stars (in principle possible almost in real time);
analysis of positional residuals (possible only in the final reductions when all observations of the same star are compared).

Being to some extent complementary, all three methods should of course be employed.

3.1. A priori information

The usefulness of existing data on double stars should not be underestimated. From the completeness factors given by Heintz (1969) and his Table I it is found that about 70% of the systems with primary magnitude $m_1 = 8$ to 9, $\Delta m < 5$, and $0.25 < \rho < 15''$ are listed in the Index Catalogue of Visual Double Stars (Jeffers et al., 1963), at least north of $\delta = -30^\circ$.

For most of these we have approximate position angles, separations, and magnitude differences. Even a modest a priori knowledge of $\Delta m$ and projected separation $d$ (in the scanning direction), corresponding to mean errors of (say) 0.5" and 0.1", will in many cases drastically improve the decomposition of the signal, or resolve possible ambiguities. Such mean errors are quite realistic for a large fraction of known double stars, considering the very slow orbital motion for most systems (the median relative projected motion is only 0".002/yr for systems with $\rho = 1$).}

3.2. Signal decomposition

Detection by decomposition of the detector signal can be regarded as a multiple-hypotheses test. Let $H_n$ be the hypothesis that there are exactly $n$ stars in the IFOV. Then $H_1, H_2, \ldots, H_{n_{\text{max}}}$ are sequentially tested against each other and the hypothesis with smallest $n$ giving satisfactory fit to the observed signal is accepted. The signal, which is a series of photon counts, is in $H_n$ modeled as an inhomogeneous Poisson process with intensity

$$N_n(x) = B + \sum_{j=1}^{n} \text{dex}(-0.4 m_j) I(x - x_j), \quad m_1 \leq m_2 \leq \ldots \leq m_n, \quad n \leq n_{\text{max}},$$

where $I(x)$ is the intensity variation with coordinate $x$ for a single star, supposed to be accurately known, and $B$, $m_j$, and $x_j$, $j = 1, 2, \ldots, n$, are $2n+1$ parameters representing the background intensity and the magnitudes and positions of the $n$ stars. Application of hypothesis $H_n$ amounts to fitting the intensity $N_n(x)$ to the observed counts (e.g. by maximum likelihood) and computing a suitable goodness-of-fit measure (such as $\chi^2$ or the likelihood itself).

The classical strategy for selecting one hypothesis is to use a test which maximizes the probability of detection (rejecting $H_1$ when it is false) while, at the same time, keeping the risk of false detection (e.g. accepting $H_2$ when $H_1$ is true) at an acceptable, pre-determined level. However, in our case one knows the a priori probability of each hypothesis and the cost of each kind of wrong decision (in terms of loss of accuracy). Then one can select the hypothesis which minimizes the expected cost of taking that decision. Although the statistical tools for doing all this are well known and simple in principle, it appears that the computational effort involved is substantial and a slightly less sophisticated but still efficient procedure ought to be developed.

As indicated in the Introduction, it is not possible to fit unambiguously any number of stars to the output signal, because there is only a limited number of Fourier components in the single-star signal $I(x)$. This very fundamental limitation is due to diffraction of the light (Fig. 4). The criterion for unambiguous decomposition is

$$n_{\text{max}} \leq 8 D/\lambda$$
Figure 4. Left: Convolution of the diffraction pattern of a star with the periodic grid transmittance results in a periodic intensity variation \( I(x) \) containing only a finite number of harmonics. With the shorter grid period \( s = 0.08 \) the signal is purely sinusoidal, while \( s = 1.4 \) yields also the first harmonic. The longer period is preferred for double-star observation, since the superposition of two such signals can be decomposed into the signals of each star. This is impossible with the shorter period, because the sum of two sinusoids is just another sinusoidal.

Right: The finite harmonic content of the output signal follows from the fact that the modulation transfer function (MTF) of the telescope is zero above a certain spatial frequency \( f_0 = D/\lambda \), where \( D \) is the aperture diameter and \( \lambda \) the wavelength. In the Fourier domain, convolution is replaced by multiplication: the Fourier transform (FT) of the output signal is the product of the FT of the diffraction pattern (which is just the MTF) and the FT of the grid transmittance, which is an infinite series of harmonics.

Figure 5. Mean error of the position of a 9th magnitude star from a 4 sec observation, as function of grid period \( s \). The double-star solution, taking into account a 12th magnitude secondary at various projected distances \( d \), is possible only for \( s > 1.1 \), when the first harmonic appears in the output signal.
in which \( s \) is the grid period in rad, \( D \) the length of the aperture normal to the grid slots, and \( \lambda \) the wavelength of light. With \( D = 16 \text{ cm} \) and \( \lambda = 430 \text{ nm} \), one finds that \( n = 1 \) and \( n = 2 \) require \( s > 0.055 \) and \( s > 1"1 \), respectively. Thus \( s < 0.055 \) gives no modulation at all and no position measurement is possible; \( 0.055 < s < 1"1 \) gives a sinusoidal modulation, while \( 1"1 < s < 1"65 \) gives also the first harmonic, etc.

Figure 5 indicates how the grid period can be optimized for double-star detection. It shows the mean error of the derived position of the program star (primary), with and without a parasitic star three magnitudes fainter. As expected, the two-star solution becomes singular for \( s < 1"1 \). The optimum period for single-star observation is approximately 0"8, but then a two-star solution is impossible. For observation of double stars, on the other hand, the optimum grid period is around 1"4, which nearly coincides with the shallow second minimum of the single-star mean error only 20% above its first minimum at 0"8. It seems very advisable to accept this 20% increase of the mean error of single-star observations in exchange for the possibility of signal decomposition. The comparatively rare cases of triple stars probably do not justify a still longer grid period of 2"3, at the third minimum of the single-star mean error.

The probability of detecting a parasite by signal decomposition has been evaluated for a number of configurations. For primary magnitude \( m_1 = 9 \), 20 sec accumulated observing time, and a detection criterion yielding a 1% probability of false detection, one finds that detection is likely (probability > 0.5) if the projected separation is at least 0"1 to 0"3 (for \( \Delta m = 0 \) to 3, respectively), but improbable for \( \Delta m > 3.5 \) at any separation.

3.3. Analysis of residuals

When solving for the position, proper motion, and parallax of a double star, not recognizing its duplicity, one should find that the positional residuals show up a certain pattern (like in Fig. 2) by which the duplicity may be detected. Thus far, it has not been studied if and how this can be done, but some indication of the sensitivity of the method is provided by the radius of dispersion (Fig. 3). Compared with a solution for a single star, the variance of the residuals should be greater by the square of the dispersion radius. Thus there should be a good chance of detection if the dispersion radius is at least 0"006, which is the expected standard deviation for single-star observations. This condition is satisfied for \( \Delta m < 3 \) and \( \rho > 0"25 \). The sensitivity is therefore about the same as for signal decomposition.

4. CONCLUSIONS

An astrometry satellite of the type considered will be able to detect and obtain consistent and accurate positions, proper motions, and parallaxes of most double stars with \( \Delta m < 3.5 \) and \( 0"15 < \rho < 15" \), the lower limit of separation depending slightly on \( \Delta m \) as indicated in Section 3.2. Not counting the known double stars, there remain about 3% undetected binaries with \( \Delta m \) between 3.5 and 5, causing errors of 0"008 or less. This is acceptable in view of the expected distribution of accidental errors.

The number of new double stars discovered by the satellite can be estimated from the known statistics of double stars and the detection criterion given above. Among the 100,000 stars of an observing program, there will be about 70,000 double or multiple stars. Only 11,500 of them are known of at present. The satellite will detect some 13,000 cases, of which 3,500 are new double stars, 9,000 are previously known
systems, and 500 are false detections. Being based on a number of assumptions, these figures must of course be taken with great caution. The position and magnitude of the secondary will be measured for all detected double stars, but generally with a precision reduced in proportion to the relative brightness of the secondary.

REFERENCES