Determining PPN-\(\gamma\) with Gaia's Astrometric Core Solution

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Gaia’s Astrometric Global Iterative Solution (AGIS)

- AGIS is the core astrometric processing of raw Gaia data in which several processes are executed iteratively until convergence
- A direct solution is impractical by many orders of magnitude
  - the number of observations ($\sim 8 \times 10^{11}$)
  - the number of unknowns (parameters) ($\sim 5 \times 10^9$)
- AGIS uses a subset of at least 100 million astrometrically well behaved ‘primary’ stars.
Source
Typically, we update 5 of 6 astrometric parameters using a weighted least squares solution for $\sim10^8$ stars.

**Attitude**
Solve for the three axis attitude as a function of time using a quaternion $q(t)$ represented by cubic splines.

**Calibration**
Estimate the instrument calibration parameters (e.g. CCD & pixel positions, BA, etc.)

**Global**
Estimate global parameters (e.g. PPN-$\gamma$)
Parallax:
\[ \Delta \theta \propto - \sin \theta \]

PPN-\( \gamma \):
\[ \Delta \theta \propto + \cot(\theta/2) \]

\( \theta \) = sun-star angle

Principle of PPN-\( \gamma \) measurement by Gaia
AGISLab

AGISLab is a **scaled down** version of AGIS so many experiments can be done using a random distribution of stars

Uses a scale parameter $S$:

- $10^6$ primary objects (instead of $10^8$)
- Longer attitude spline knot interval
- Wider fields of view (vary focal length)
- Scan rate slowed down

Scaling keeps constant:

- $N \propto S$
- $\tau \propto S^{-1}$
- $F \propto \sqrt{S}$
- $\omega \propto \sqrt{S}$

<table>
<thead>
<tr>
<th>$S$, scale factor</th>
<th># primary stars</th>
<th>knot sequence [sec]</th>
<th>Focal length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1 \times 10^6$</td>
<td>15</td>
<td>35.0</td>
</tr>
<tr>
<td>0.05</td>
<td>$5 \times 10^4$</td>
<td>300</td>
<td>7.8</td>
</tr>
<tr>
<td>0.01</td>
<td>$1 \times 10^4$</td>
<td>1,500</td>
<td>3.50</td>
</tr>
</tbody>
</table>

100 $\mu$as Gaussian observation noise $\sim 11.8$ $G_{\text{mag}}$ stars
Gravitational Light Deflection in AGISLab

AGISLab now includes a simple model of light bending due to the Sun.

\[ \hat{u} = \left( \hat{u} + h \frac{(1 + \gamma)GM_s c^{-2}}{h(h + \hat{u} \cdot h)} \right) \quad \text{and} \quad \frac{\partial \hat{u}}{\partial \gamma} = \left( h - \hat{u}(\hat{u} \cdot h) \right) \frac{GM_s c^{-2}}{h(h + \hat{u} \cdot h)} \]

where \( \hat{u} \) is the star direction and \( h \) is the heliocentric coordinate of Gaia.

At 90° to the Sun gravitational light bending is \(~4000 \mu as\).

The effects of other solar system bodies are not included.

Combining millions of measurements can determine light bending to unprecedented accuracy via PPN-\( \gamma \).
Calculating PPN-$\gamma$

A new algorithm in AGISLab

• Initially PPN-$\gamma$ was calculated but convergence was slow
• Slow convergence was caused by a correlation with the parallax zero point
• To solve this we introduced an extra pseudo parameter, equivalent to a common parallax shift but constrained to zero

$$\frac{\partial F}{\partial G[1]} = \frac{\partial F}{\partial \pi} = \frac{\partial F}{\partial (\pi + \pi_0)}$$

and $G[1]$ is reset to zero each iteration

• This new pseudo parameter eliminates the correlation and also allows it to be estimated
• When the correlation is eliminated we get improved convergence
An example using 50000 random stars ($S=0.05$) and 100 $\mu$as observation noise

Solving for only PPN-$\gamma$-1 gives slow convergence due to a strong correlation with the parallax zero point.

Solving for PPN-$\gamma$ with the artificial pseudo parameter constraint to zero accelerates convergence automatically.

Correlation between PPN-$\gamma$ and the parallax zero point is $\sim 91\%$ which affects the accuracy of PPN-$\gamma$. 
Preliminary results

Running 100 simulations with 10000 stars (S=0.01) allows the distribution of the errors in $PPN-\gamma$ to be calculated.

We must correct the total weight of observations in all magnitude bins to give a realistic estimate for Gaia.

Extrapolation to large scale simulations (ESAC-AGIS & Gaia) is then possible.

Pessimistic estimate: $\sigma_{\gamma} = 4.01 \times 10^{-6}$

Optimistic estimate: $\sigma_{\gamma} = 1.27 \times 10^{-6}$

The optimistic estimate is not trivial to achieve but is possible!

$$\sigma_{\gamma}^{AGIS} = M_{Deviation} \times \sigma_{\gamma}^{AGISLab} \times \left( \frac{W_{AGIS}}{W_{AGISLab}} \right)^{-\frac{1}{2}}$$

$$M_{Deviation} = \sqrt{\frac{1}{N} \sum_{N} \left( \frac{\gamma - 1}{\sigma} \right)^2} = 1.285 \pm \frac{1.285}{\sqrt{2N}}$$
Basic Angle Variations

- The basic angle (BA) monitor on Gaia will detect BA variations accurate to 0.5 \( \mu \text{as} \).
- BA data will be made available every 5 minutes to be used for calibration.
- Random errors with these characteristics are added to the simulation of observations to assess their impact on the solution for PPN-\( \gamma \).

<table>
<thead>
<tr>
<th></th>
<th>True Observations ( \gamma - I )</th>
<th>Noisy Observations ( \gamma - I )</th>
<th>Noisy Observations ( \Delta \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA noise (S=0.05)</td>
<td>-3.129( \times 10^{-7} )</td>
<td>-5.743( \times 10^{-6} )</td>
<td>-3.131( \times 10^{-7} )</td>
</tr>
<tr>
<td>No BA noise (S=0.05)</td>
<td>0.134( \times 10^{-9} )</td>
<td>-5.429( \times 10^{-6} )</td>
<td></td>
</tr>
</tbody>
</table>

- This tells us that the BA variations are additive but will not really affect the solution.
- However, this assumes that there are no systematic errors in the calibration.
How to improve the PPN-γ result

- Clearly selecting extra primary stars will improve the accuracy of PPN-γ
- However we don’t need 10 times as many stars to improve the result by a factor of ~3.2

$$\sigma^\text{AGIS}_\gamma \propto 1/\sqrt{N_{\text{Obs.}}} \quad \text{and} \quad \sigma^\text{AGIS}_\gamma \propto W^{-1/2}_{\text{AGIS}}$$

- The weight of observations is clearly much stronger from 12th to 16th magnitude
- The total number of stars in this magnitude range is not terribly large < 94 million
- If the number of these stars are increased in AGIS then $\sigma_\gamma$ could be improved without significantly increasing the total number of stars
Conclusions

• Previous estimates of the accuracy of PPN-γ were based on extrapolating Hipparcos results to Gaia, assuming that all observations could be used in the solution.

• Since then the revised Gaia design leads to larger estimated errors for PPN-γ.

• It is desirable to increase the total weight of observations in AGIS by selecting as many primary stars as possible in the appropriate magnitude range.

• The total number of observation used in AGIS does not need to significantly increase.

• These preliminary results need to be verified by full scale AGIS simulations - this is ongoing and preliminary results agree with our predictions.
Actual errors

Accelerated convergence

Updates

Astrometric parameters, for 50000 sources ($S=0.05$)

Attitude parameters (scale parameter=0.05, knot interval=300.0 sec)

Parallax errors in iteration 50.0