Stellar Intensity Interferometry:  
Imaging capabilities of air Cherenkov telescope arrays

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\textbf{Abstract}

Sub milli-arcsecond imaging in the visible band will provide a new perspective in stellar astrophysics. Even though stellar intensity interferometry was abandoned more than 40 years ago, it is capable of imaging and thus accomplishing more than the measurement of stellar diameters as was previously thought. Various phase retrieval techniques can be used to reconstruct actual images provided a sufficient coverage of the interferometric plane is available. Planned large arrays of Air Cherenkov telescopes will provide thousands of simultaneously available baselines ranging from a few tens of meters to over a kilometer, thus making imaging possible with unprecedented angular resolution. Here we investigate the imaging capabilities of arrays such as CTA or AGIS used as Stellar Intensity Interferometry receivers. The study makes use of simulated data as could realistically be obtained from these arrays. A Cauchy-Riemann based phase recovery allows the reconstruction of images which can be compared to the pristine image for which the data were simulated. This is first done for uniform disk stars with different radii and corresponding to various exposure times, and we find that the uncertainty in reconstructing radii is a few percent after a few hours of exposure time. Finally, more complex images are considered, showing that imaging at the sub-milli-arc-second scale is possible.

1. INTRODUCTION

There has been a recent interest in the revival of Stellar Intensity Interferometry (SII) due to the excellent baseline coverage of planned air Cherenkov telescope arrays.\textsuperscript{1,2} This interest has lead to developments in instrumentation, experimentation, and simulations of the capabilities of this technique. Various analog and digital correlator technologies\textsuperscript{3} are being implemented by LeBohec et. al.,\textsuperscript{4} and cross correlation of streams of photons with nanosecond scale resolution has already been achieved. The suitability of various proposed array configurations is being evaluated by Jensen et al.\textsuperscript{5} to understand their different sensitivities for interferometric imaging before final choices of the array layouts are made. Image reconstruction algorithms such as the one suggested by Holmes et. al.\textsuperscript{6} have opened the possibility of imaging and have become interesting subjects in their own right. In this paper we will focus on the imaging capabilities and limitations of air Cherenkov telescope arrays used as high angular resolution intensity interferometers.

High angular resolution astronomy in the optical range will open a whole new field of exploration. The possibility of viewing stars as extended objects will enable the testing of many current astrophysical models, and the knowledge acquired will have consequences on many fields related to stellar astrophysics. As a first example consider the measurement of stellar diameters, which can be performed to an accuracy of a few percent with the methods discussed in this paper (see section 4.1). When measuring diameters at different wavelengths, we can learn about the behaviour of the optical depth as a function of the radius of a particular star.\textsuperscript{7} This type of measurement becomes particularly useful at shorter wavelengths ($\lambda \sim 400\text{nm}$) than those feasible with conventional amplitude (Michelson) interferometry. Another interesting science case is the study of fast rotating stars, for which we can measure oblateness, pole brightening, and disk formation. There is also the study of

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interacting binaries. An actual image of an interacting binary will not only aid in the determination of the orbital parameters to a high degree of accuracy, but also in the study and imaging of mass transfer and accretion. Although not rigorously discussed in this paper, imaging mass transfer will improve our understanding of late stellar evolution, i.e. the formation of type II supernovae, which are of great importance in cosmology, and our understanding of the formation of compact objects. The number of interesting science cases and astrophysical targets is overwhelmingly large, and a detailed discussion is given by Dravins et al.

We propose the use of Intensity Interferometry for high angular resolution astronomy (see Holder et. al. for more details). This technique was introduced in the 1950’s by R. Hanbury Brown and implemented in the 1960’s with the Narrabri Stellar Intensity Interferometer, accomplishing the measurement of over thirty stellar diameters. The use of planned air Cherenkov telescope arrays poses a unique opportunity to revive Intensity Interferometry. With hundreds of telescopes separated by up to ~ 1 km, it will be possible to have an unprecedented coverage of the Fourier plane and thus achieving sub-milliarcsecond resolution.

The most important difference between SII and amplitude interferometry is that SII relies on the correlation between the low frequency intensity fluctuations and so does not rely on the relative phase of the individual waves received at different telescopes. Intensity interferometry measures the squared modulus of the complex degree of coherence $|\gamma|^2$.

$$|\gamma|^2 = \frac{<\Delta I_i \Delta I_j>}{<I_i><I_j>}$$ (1)

Here $<I_i>$ is the time average of the intensity received at a particular telescope $i$, and $\Delta I_i$ refers to the low frequency intensity fluctuations received at telescope $i$. Intensity interferometry has several advantages and disadvantages when compared to amplitude interferometry. The main advantages are that it is insensitive to atmospheric turbulence and that it does not require high optical precision. The complex degree of coherence is proportional to the Fourier transform of the object in the sky (Van Cittert-Zernike theorem), and since we measure the modulus squared of $\gamma$, the main disadvantage is that the phase of the Fourier transform is lost in the measurement process, posing difficulties in recovering actual images from magnitude information only. In addition to imaging difficulties, measuring a second order effect also results in sensitivity issues, which can be dealt with by using large light collection areas and exposure times. As for the imaging limitations, several phase retrieval techniques exist, and we will implement a two dimensional version of the one dimensional approach introduced by Holmes & Belen'kii. It is important to note that our results pertain to a single phase recovery algorithm, and a comparison to other algorithms is currently being investigated. Once a sufficient coverage of the Fourier plane is available, and phase recovery is performed, a study on imaging capabilities can be performed. Here we will first concentrate on the study of the uncertainty when reconstructing disk-like stars. Then a less exhaustive analysis on the capabilities is performed for more complicated images such as oblate rotators, binary stars and stars with bright & dark features.

The outline of the paper is the following: First we will briefly discuss the phase retrieval technique. Then we will discuss the simulation of our data and how it will be fitted to an analytic function so that the phase retrieval method can be applied. Finally we will discuss the capabilities for imaging disk-like stars, binary stars and more complicated objects.

2. PHASE RECONSTRUCTION

The objective of phase reconstruction is to recover the phase of the Fourier transform of the image from magnitude information only. The resulting image is then reconstructed up to an arbitrary translation and reflection. It is simpler to first understand phase retrieval in one dimension and then generalize to two dimensions. One possible route towards phase retrieval starts by first approximating the continuous Fourier transform $I(x)$ by

\footnote{For a more detailed discussion on the advantages of Intensity Interferometry see Hanbury Brown}
a discrete one \( (I(m\Delta x) = \sum_j O(j\Delta \theta)e^{ijm k_0 \Delta x \Delta \theta}, \) where \( O(\theta) \) is the image in the sky and \( k_0 \) is the usual wave vector). Then the discrete Fourier transform can be expressed as a magnitude times a phasor \( (I(z) = R(z)e^{i\Phi(z)} \) where \( z \equiv e^{im k_0 \Delta x \Delta \theta} \) is complex). The most important step is then to apply the theory of analytic functions i.e. the Cauchy-Riemann equations. These relate the phase \( \Phi \) and the log-magnitude \( \ln R \) along the real or imaginary axes. One can show by using the Cauchy-Riemann equations, that the phase differences along the radial direction in the complex plane are directly related to the differences in the logarithm of the magnitude (see Holmes & Belen’kii for more details), so that integrating the Cauchy-Riemann equations directly does not immediately solve for the phase. In other words, phase differences along the purely real or imaginary axes are not available directly from the data.

Since \( z \), the independent variable of the Fourier transform \( (z \equiv e^{im k_0 \Delta x \Delta \theta}) \), has modulus equal to 1, the phase differences that we seek lie along the unit circle in the complex plane. Consequently, the procedure to find the phase consists in first assuming a plausible solution form, then taking differences in the radial direction of the complex plane, and finally fitting the data to the radial differences of the assumed solution. A general form of the phase can be postulated by noting that the phase is a solution of the Laplace equation in the complex plane (applying the Laplacian operator on the phase and using the Cauchy-Riemann equations yields zero). Since the phase differences are known along the radial direction in the complex plane we can take radial differences of the general solution and then fit the log-magnitude differences (available from the data) to the radial differences of the general solution.

One can think of this one-dimensional reconstruction as a the phase estimation along a single slice in the Fourier plane. A generalization to two dimensions can be made by doing the same procedure for several slices. The direction of the slices is arbitrary, however for simplicity we reconstruct the phase along horizontal or vertical slices in the Fourier plane, and noting that one can relate all slices with a single orthogonal slice, i.e. once the phase at the origin is set to zero, the single orthogonal slice sets the initial values for the rest of the slices. The resulting reconstructed phase will be arbitrary up to a constant and a linear term, which corresponds to a translation. It should be noted that the above solution approach gave reasonably good results. However, it is not the only possible approach. We have also investigated Gerchberg-Saxton phase retrieval, Generalized Expectation Maximization, and other variants of the Cauchy-Riemann approach.

3. PROCEDURE

Having briefly discussed the phase reconstruction algorithm, our basic procedure for recovering images is the following: First we simulate realistic data as would be obtained from an air Cherenkov telescope array such as CTA or AGIS. Once simulated data are available, they are fitted to an analytic function so that the phase recovery algorithm can be applied in a more straightforward way. Finally, once the phase is recovered, the inverse Fourier transform will provide us with a reconstructed image. Some details concerning the simulations and data fitting approach now follow.

3.1 Simulation of realistic data and array design used

The simulation of the the data that may be produced by an array of telescopes starts from a pristine image, generally 2048 × 2048 pixels with an arbitrary dynamic range. The squared magnitude of the Fourier transform of this image is obtained by means of an FFT algorithm and it is normalized so its maximum at zero baseline is equal to one. With a wavelength \( \lambda = 400\, nm \), and the full scale of the image typically set to 10 mas, this provides a value for the expected degree of coherence on a square grid with a pitch of \( \sim 8.2\, m \) extending over a \( \sim 16.8\, km \times 16.8\, km \) area.

The squared Fourier magnitude map is sampled by the set of pairs available in the simulated array. Simulations presented here have been obtained with an array of \( N = 97 \) telescopes, each with a light collecting

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\(^\dagger\)The C-R equations can be applied because \( I(z) \) is a polynomial in \( z \), where \( z \equiv e^{i\theta} \).

\(^\ddagger\)If \( \xi \) is the real axis and \( \psi \) is the imaginary axis, then a difference along the radial direction is \( \Delta \xi + i\Delta \psi \).
Figure 1. Array configuration used for our analysis.

Area of 100 m² and a quantum efficiency \( \alpha = 0.30 \) resulting in an effective area of 30 m². The telescopes are distributed in the field according to an early design of the CTA array shown in figure 2. Such an array provides a coverage of the interferometric \((u,v)\) plane with \( N(N-1)/2 = 4656 \) baselines many of which are redundant. The baselines are shown in figure 2. The degree of coherence recorded by each baseline is obtained from a linear interpolation between the closest four points in the Fourier magnitude table.

The data recorded by a real array would be affected by the diurnal motion of the observed star which affects the effective baselines by projection. The average correlation must then be recorded for each baseline at time intervals short enough for the baseline change to be negligible. In the first simulation study reported here we have decided to avoid this complication and simulate data that would result from the observation of fixed stars at zenith so the effective baselines used are those shown in figure 2 without any further projective distortion. The implications of this simplification choice are a less uniform sampling of the \((u,v)\) plane compensated by smaller error bars on the degree of coherence from each baseline record. These two effects essentially cancel each other as long as small scale features in the \((u,v)\) plane are not central to the analysis. The benefits from the simplification is a reduction in the volume of data to handle (each simulation produces a single record for each pair of telescopes) and eliminates further arbitrary parameters (such as the site latitude, celestial declination, range of hour angles and time interval between recordings).

Once the degree of correlation within each baseline has been obtained, Gaussian noise is added. The Gaussian nature of the noise was tested with detailed simulation of a pair of photo-multiplier tube signals corresponding to a random stream of photons. The time integrated product of the two traces was Gauss distributed. The magnitude \( m_V \) of the star is used to compute a spectral density \((m^{-2}s^{-1}Hz^{-1})\) according to \( n = 5 \times 10^{-5} \times 2.5^{-m_V} \). The standard deviation of the Gaussian noise added to the pair of telescopes \((i,j)\) is calculated as \( \sigma = n \sqrt{A_i \cdot A_j \cdot \Delta f \cdot \Delta t / 2} \) where \( A_i \) is the effective light collection area of the \( i^{th} \) telescope, \( \Delta f \) is the signal band-width and \( \Delta t \) is the observation duration. For simulations presented in this paper \( \Delta f = 200 MHz \) which is a realistic choice when considerations on air Cherenkov telescope optics and electronics are taken into account (See Holder & LeBohec\(^1\) and LeBohec et al.\(^4\)).
3.2 Fitting the data to an analytic function

The phase reconstruction algorithm is greatly simplified when data are known on a square grid rather than in a ‘randomly’ sampled way as is directly available from observations. This is because sampling data on a fine square grid enables an easier estimation of the derivatives of the log-magnitude.

Assuming that data \( f(x_i) \) are known at positions \( x_i \), with uncertainty \( \delta f(x_i) \), our goal is then to find a function that minimizes the following \( \chi^2 \)

\[
\chi^2 = \sum_i \left[ \frac{f(x_i) - \sum_k a_k g_k(x_i)}{\delta f(x_i)} \right]^2.
\]

Here, the \( a_k \)'s are the coefficients of the basis functions \( g_k \) that we want to use to fit our simulated data. Any complete basis will suffice in theory, however it is more appropriate to choose a set of basis functions that tend to zero at infinity. The reason for this requirement of our basis functions is so that data are more realistically fitted in regions where there is not much data available. For the case of the CTA array design that we used, we noticed that there is less data at baselines greater than 600 m (see figure 2). We found that basis functions that meet this requirement are the solutions to the two dimensional quantum-mechanical harmonic oscillator, i.e. Hermite polynomials with Gaussian envelopes. These also turn out to be convenient because they are eigenfunctions of the Fourier transform operator.

Now we can turn this problem into a linear system by taking derivatives with respect to the unknowns \( a_k \). Our data fitting typically starts with a small number of basis elements, then we check to see if a certain reduced \( \chi^2 \) is met (in our case, we chose an acceptable reduced \( \chi^2 \) to be 1.5), if this condition is not met, then the number of basis elements is increased iteratively until it does.

4. IMAGING CAPABILITIES

We may start by quantifying the resolution of an air Cherenkov telescope array such as the one shown in figure 2 by recalling that quantities that are related to each other by a Fourier transform obey an uncertainty relation. For an order of magnitude estimation it suffices to relate the size of the array to the maximum resolution by

\[
\Delta \theta \sim \frac{\lambda}{\Delta x}.
\]

Figure 2. Available baselines for the array design used in this study.
With a kilometer size array ($\Delta x \sim 1\ km$), and a wavelength of $\lambda \sim 400\ nm$ we obtain a resolution of $\Delta \theta \sim 0.1\ mas$. On the other hand, the largest objects observable with an array whose inter-telescope separation is of the order of $\Delta x \sim 50m$ is $\Delta \theta \sim 1\ mas$. These order of magnitude considerations will be taken into account when performing simulations and image reconstructions, i.e. the minimum and maximum size of pristine images will not go far beyond these limits.

We tested the imaging capabilities for simple objects, namely uniform disk-like stars, oblate rotating stars, binaries, and more complex images. First we will concentrate on the capabilities and limitations for reconstructing uniform disk-like stars. We will show that such a preliminary analysis reveals more precisely, when compared to the previous estimate, the sizes of objects that can be observed. Even though using a Cauchy-Riemann based approach to recover images might not be the most efficient way to measure stellar radii, such a study will start to quantify the abilities of measuring other scale parameters in more complicated images (oblateness, distance between binary components, etc.).

4.1 Uniform disks

Simulated data were generated corresponding to uniform disk stars of a particular brightness. We set the brightness to magnitude 6 after noting that an error of a few percent in the simulated data can be achieved in a few hours. Also, $6^{th}$ magnitude stars are appropriate since they roughly correspond to the upper limit for most of the interesting astrophysical targets found by Dravins et al.$^9$

It is interesting to first study the uncertainty for a particular exposure time and a brightness corresponding to $6^{th}$ magnitude. We simulated data corresponding to uniform disks of random radii up to 1$\ mas$ for 50 hours of exposure time. An example of such a reconstruction is shown in figure 3b, where the brightness is shown in arbitrary units between 0 and 1. A first look at the example reconstruction reveals that the edge of the reconstructed disk is not sharp, so a threshold in the brightness was applied for for measuring the radius. The radius was measured by counting pixels above a threshold and noting that the area of the disk is proportional to the number of pixels passing the threshold. After experimenting for different radii, we chose the threshold for measuring the radius to be 0.2. We can now compare the simulated and reconstructed radii as shown in figure 3a, where each point in the figure corresponds to an individual simulation (including noise) and reconstruction.

Figure 3a clearly shows that stellar diameters ranging from 0.05 mas to 0.5 mas can be measured with uncertainties smaller than 5%. The ‘tail’ seen in the bottom left of figure 3a shows the smallest measurable radius, so we take this radius ($\approx 0.03\ mas$) to be the point spread function (PSF) of the array. The uncertainty shown in the sub-figure in figure 3 was estimated after running noisy simulations many times as is shown in figure 4.

It can be seen from figure 3a, that the uncertainty increases roughly linearly as a function of the pristine (simulated) radius. This can be understood from the following argument: As the pristine radius decreases, the distance to the first zero in the correlation increases as $\sim r$, so the number of telescopes contained within the Airy disk increases as $\sim r^2$. Consequently, decreasing the pristine radius is equivalent to increasing the number of independent measurements by a factor of $\sim r^2$ at most (at least by a factor of $r$). Since the uncertainty will decrease as the square root of the number of independent measurements, the error will decrease as $\sim r$ at most. For radii above 0.6$\ mas$, there are simply not enough baselines to constrain the Fourier plane information for image reconstruction. For radii greater than 0.6$\ mas$, the size of the Airy disk is of the order of 100$\ m$, and this results in having less than 100 independent measurements as can be seen in figure 2. Also shown in figure 4 is the percent error as a function of time for two different radii, where it can be seen that a percent error of less than 5% is achieved after only a few hours.

5. COMPLEX IMAGES

Our algorithm has also been tested on more complex images such as oblate rotating stars, binary stars, and stars with brighter or darker regions. Since we have not developed a proper tool for quantitatively comparing
Figure 3. a) Simulated vs. Reconstructed radii for magnitude 6 stars with 50 hours of observation time. The top sub-figure shows the uncertainty for a 0.2 mas measurement. The bottom sub-figure shows the residual (Reconstructed-Real) along with the uncertainty in the radius. b) Example of a reconstructed uniform disk of radius 0.2 mas. Also shown is a slice of the reconstructed image (solid line) compared to a slice of the pristine image (dashed line).

Figure 4. a) Histogram of real radius minus reconstructed radius for 50 hours of exposure time on a 6th magnitude star of 0.2 mas radius. b) Percent error as a function of time for several reconstructed radii.
simulated and reconstructed images, we will only show a few representative examples of what the algorithm is capable of. Figure 5a shows an example reconstruction of an oblate rotating star of magnitude 3 and 10 hours of observation time, the semi-major axis and semi-minor axis of the pristine ellipse are 0.2 mas and 0.12 mas respectively. Also shown in this figure, and all subsequent ones, is the pristine image convolved with the point spread function of the array (see section 4.1). Reconstructed noise of less than 10% can be seen in the figure. These noise fluctuations are not so much a consequence of noise in the simulated data, but of the reconstruction algorithm itself, and the preferential direction of this reconstructed noise (along the vertical direction) is due to our choice of the slice direction for the phase reconstruction, i.e. the phase was reconstructed by taking horizontal one-dimensional slices of the magnitude, and then related to each other with a single vertical slice. As for the structure (bumps) within the star in figure 5a, these start to appear when either the star becomes bright or enough exposure time is supplied so that information other than the first lobe in the Fourier magnitude is significant. In other words, when high frequency portions are visible in the Fourier magnitude, fictitious structure starts to become visible. This is most likely due to the fact that most of the high frequency information in the Fourier plane is used to reconstruct a dark background of several mas's with a central bright region.

The case of a binary star is shown in figure 5b, corresponding to a magnitude 6 binary star for 12 hours of exposure time. The noise in the reconstructed image has the same origin as in the case of the oblate star. Although the inclination angle was well reconstructed in both cases in figure 5, image reconstruction is degraded when the symmetry axis is neither the x or y axis. Again, this is due to the particular phase recovery method of taking horizontal or vertical slices and the degradation is significantly reduced when aligning one symmetry axis of the magnitude to our x or y axis.

Having this symmetry consideration in mind, a slightly more complicated example is shown in figure 6a, corresponding to a star obscured by a disk (of dust for example). A black streak in the pristine image representing the obscuring disk is aligned with the z axis. The black streak can be easily seen in the reconstruction as well as the contour of the obscured star. This image becomes increasingly easier to reconstruct as the image becomes more and more symmetric, that is, as the black streak in the pristine image moves towards the center.

\*Since the simulated and reconstructed objects can differ by translations and reflections, developing a tool than can accurately quantify the difference between simulated and reconstructed images is not trivial.
Figure 6. a) Simulated and reconstructed star obscured by a disk. This corresponds to magnitude 4 and 15 hours of observation time. b) Uniform disk of radius 0.2 mas with a dark spot of radius 0.05 mas. The simulation was done for magnitude 6 and 100 hours of observation time.

of the star\(^\text{II}\). As a final example, we considered the case of a dark spot in a star as shown in figure 6b. The simulation corresponds to a magnitude 6 star and 100 hours of observation. Even though the reconstructed image appears brighter in the bottom, and the location of the spot is more symmetric, the size of the spot (comparable to the PSF) is well reconstructed.

6. CONCLUSION AND OUTLOOK

We have shown that planned air Cherenkov telescope arrays have sufficient baselines to provide an excellent coverage of the (u,v) plane. By first simulating realistic data, we show that it is possible to achieve a signal to noise ratio of the order of 10 within a few hours for relatively faint stars (magnitude 6), and obviously higher S/N for brighter stars. Once data were simulated we also show that imaging is possible using a Cauchy-Riemann phase recovery technique. A study of the error propagation with disk-like stars reveals that the uncertainty in reconstructed radii is of a few percent. We also explored imaging capabilities for more complex images such as oblate rotating stars, binary stars, and stars with dark/bright regions yielding good results. A quantitative analysis of the reconstruction capabilities for complex images is still in progress.

The array design could certainly be improved by including telescopes at shorter distances. This could significantly improve the size range of observable objects, in particular, we could observe objects of more than 1 mas across at 400 nm.

The analysis can also be improved by doing several things: If the pristine object has a symmetry axis, a first reconstruction can be made to find it, and then a second image reconstruction can be made to improve the first results. Something else worth implementing is a first reconstruction only constraining the low frequency components of the phase by using the low frequency part of the magnitude, then a second reconstruction could be performed, only dealing with the internal details of the image.

\(^\text{II}\)This can be understood by noting that the Fourier magnitude of a non symmetric binary becomes almost indistinguishable from the one corresponding to a central bright star with two fainter companions at either side.
To conclude this simulation phase, pristine images generated from astrophysical models should be generated in order to identify how much of the astrophysical model can actually be constrained. This aspect of the simulation phase is currently under development.

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