P5: The local mass density

Purpose

The purpose of this project is to estimate the local mass density ρ_0 (expressed in M_S pc^{-3}) from the equation:

$$4\pi G\rho_0 = -\sigma_{ww}^2 \frac{\partial^2 \ln n}{\partial z^2} \tag{1}$$

where $n(z)$ is the number density of some tracer population as function of height (z) above the galactic plane, and σ_{ww} is the velocity dispersion along the z axis of the same tracer population.

Selection of stars

A suitable tracer population brighter than the completeness limit of the Hipparcos Catalogue is selected by taking a certain interval in colour index $B-V$ and introduce a corresponding limit in parallax ($p_{\min}$) to ensure catalogue completeness. The colour interval should be such that the number density $\nu(z)$ falls off significantly over the z interval included by the distance limit. (A good interval may be $B-V = 0$ to 0.1.)

Method

The velocity dispersion σ_{ww} is estimated as in P4 (or it can be taken directly from that project if the colour selection is the same). The main difficulty is to estimate the quantity $Q = \partial^2 \ln n/\partial z^2$ in Eq. (1). The proposed method is to divide the sphere $r \leq r_{\max}$ (where $r_{\max} = 1000/p_{\min}$) into horizontal layers of constant thickness, e.g. $\Delta z = 10$ pc, count the number of stars in each layer and divide by the volume[1] to get $n(z)$ (at the mean z coordinate of the layer). Then plot $\ln n$ as function of z and fit a parabola,

$$\ln \nu(z) \simeq c_1 z^2 + c_2 z + c_3 \tag{2}$$

e.g. using MATLAB's `polyfit` function. Then $Q = 2c_1$.

[1] The layer defined by $r \leq r_{\max}$ and $z_1 \leq z \leq z_2$ has the volume $\pi r_{\max}^2(z_2 - z_1) - \pi(z_2^3 - z_1^3)/3$ (provided that $|z_1| \leq r_{\max}$ and $|z_2| \leq r_{\max}$).