P4: Velocity dispersion and asymmetric drift

Purpose

In P1, we derived the mean space velocity of nearby stars, reflecting the motion of the sun with respect to these stars. In this project we derive further kinematic parameters for samples of nearby stars selected by distance and photometric criteria, and study their dependence on factors such as the stellar ages. For each sample, the parameters of interest are:

- the mean velocity $\langle \boldsymbol{v} \rangle = (\langle v \rangle_1, \langle v \rangle_2, \langle v \rangle_3)$ relative to the Sun;
- the velocity dispersions σ_1, σ_2, σ_3 along the principal galactic axes.

Different age groups (selected by colour) show different mean velocities and dispersions. According to theory, $\langle v \rangle_1$ and $\langle v \rangle_3$ should be roughly the same for all age groups, while $\langle v \rangle_2$ is expected to vary linearly with σ_1^2 (from the asymmetric drift equation). Extrapolation of $\langle \boldsymbol{v} \rangle$ to $\sigma_1^2 = 0$ yields $\boldsymbol{v}_{\text{LSR}}$, the velocity of the Local Standard of Rest (LSR) relative to the Sun.

Note that the first part, the determination of the mean velocity, was done already in P1 and the same code can be used here but should be applied to a wider range of stellar samples (to cover all ages).

Selection of stars

The selection of stars is made principally according to colour index $B - V$, apparent magnitude V and parallax p, as in P1. It is important that the maximum distance is adjusted for volume completeness (see P1). The whole colour range should be covered by the different samples.

Theory

Let $\boldsymbol{v}_i$ $(i = 1 \dots n)$ be the space velocities of the n stars in a group. The *mean velocity* is defined as

$$\langle \boldsymbol{v} \rangle = \frac{1}{n} \sum_{i=1}^{n} \boldsymbol{v}_i \qquad (1)$$

The *peculiar velocities* of the stars are

$$\boldsymbol{\Delta v}_i = \boldsymbol{v}_i - \langle \boldsymbol{v} \rangle \tag{2}$$

and the *velocity dispersion* along axis $j = 1, 2, 3$ is

$$\sigma_j = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left[(\boldsymbol{\Delta v}_i)_j \right]^2} \tag{3}$$

Since radial velocities are not available, we must base the calculation of $\langle \boldsymbol{v} \rangle$ and σ_j on the tangential velocities $\boldsymbol{v}_{ti} = \boldsymbol{T}_i \langle \boldsymbol{v} \rangle_i$, where $\boldsymbol{T}_i = \boldsymbol{I} - \boldsymbol{u}_i \boldsymbol{u}_i^T$ is the tangential projection matrix.

For convenience we use hereafter the notation $\boldsymbol{\tau}$ for the tangential velocity $\boldsymbol{v}_t$.

The calculation of $\langle \boldsymbol{v} \rangle$ was covered in P1; the result was:

$$\langle \boldsymbol{v} \rangle = \Big[\langle \boldsymbol{T} \rangle \Big]^{-1} \langle \boldsymbol{\tau} \rangle \tag{4}$$

To estimate the velocity dispersions is rather more tricky. We use the tangential component of the peculiar velocity:

$$\boldsymbol{\Delta \tau}_i \equiv \boldsymbol{T}_i \, \boldsymbol{\Delta v}_i = \boldsymbol{\tau}_i - \boldsymbol{T}_i \, \langle \boldsymbol{v} \rangle \tag{5}$$

Its component along axis $j = 1, 2, 3$ is:

$$(\boldsymbol{\Delta \tau}_i)_j = \sum_{k=1}^{3} (\boldsymbol{T}_i)_{j,k} \, (\boldsymbol{\Delta v}_i)_k \tag{6}$$

If we assume that $\langle (\boldsymbol{\Delta v}_i)_j \, (\boldsymbol{\Delta v}_i)_k \rangle = \sigma_j^2 \delta_{jk}$ (where δ_{jk} is Kronecker's delta), then we find by squaring (6) and taking the mean value:

$$\Big\langle \left[(\boldsymbol{\Delta \tau}_i)_j \right]^2 \Big\rangle = \sum_{k=1}^{3} \Big\langle \left[(\boldsymbol{T}_i)_{j,k} \right]^2 \Big\rangle \, \sigma_k^2 \tag{7}$$

This shows that the dispersions can be computed from

$$\begin{bmatrix} \sigma_1^2 \\ \sigma_2^2 \\ \sigma_3^2 \end{bmatrix} = \boldsymbol{S}^{-1} \, \boldsymbol{d} \tag{8}$$

where the elements of the 3×3 matrix $\boldsymbol{S}$ are (for $j = 1, 2, 3$ and $k = 1, 2, 3$):

$$S_{j,k} = \frac{1}{n} \sum_{i=1}^{n} \left[(\boldsymbol{T}_i)_{j,k} \right]^2 \tag{9}$$

and the elements in the 3×1 matrix $\boldsymbol{d}$ are (for $j = 1, 2, 3$):

$$d_j = \frac{1}{n} \sum_{i=1}^{n} \left[(\boldsymbol{\Delta \tau}_i)_j \right]^2 \tag{10}$$

Analysis of results

Based on the combined results of all students, discuss in the report:

- the variation of the mean velocity components with age;
- the variation of the velocity dispersion with age (a log-log plot may be instructive);
- extrapolation of the mean velocity to zero dispersion (some points need to be excluded).

TABLE 1. Summary of derived solar motion, velocity dispersions and maximum ages as functions of colour index $B-V$.

$B-V$ [mag]	$p_{\min}$ [mas]	n [–]	v_{s1} [km/s]	v_{s2} [km/s]	v_{s3} [km/s]	σ_1 [km/s]	σ_2 [km/s]	σ_3 [km/s]	Age [Myr]
< -0.05									
-0.05 to 0.0									
0.0 to 0.1									
0.1 to 0.2									
0.2 to 0.3									
0.3 to 0.4									
0.4 to 0.5									
0.5 to 0.6									
0.6 to 0.7									
0.7 to 0.8									