P3: Galactic orbits

Purpose

The aim of this lab is to compute the trajectories of stars within the Galaxy. In particular, the orbits of the sun and some other stars that are currently in the solar neighbourhood will be computed.

Theory

The Galactic potential

For the galactic potential we use the model described in the Lecture Notes, Sect. 7.2 (equations 7.5 and 7.6). The equations of motion are given by (7.8). In order to do the numerical integration in MATLAB, it is necessary to derive first the analytical expressions for the partial derivatives $\partial\psi/\partial x$, $\partial\psi/\partial y$, and $\partial\psi/\partial z$. Since the potential is given in cylindrical coordinates (R, ϕ, z), we use the chain rule to get

$$\frac{\partial\psi}{\partial x} = \frac{\partial\psi}{\partial R}\frac{\partial R}{\partial x} + \frac{\partial\psi}{\partial\phi}\frac{\partial\phi}{\partial x} \tag{1}$$

where the second term vanishes because our potential is axisymmetric ($\partial\psi/\partial\phi = 0$). To evaluate the first term, use that $R^2 = x^2 + y^2$.

Numerical integration

The numerical integration of the coupled differential equations (7.8) is done by considering the vector $\boldsymbol{s}(t) = (x, y, z, u, v, w)$ as function of time. The problem can then be cast in the standard form

$$\dot{\boldsymbol{s}} = f\,(t, \boldsymbol{s}) \tag{2}$$

where f is a known function. The initial conditions are given by $\boldsymbol{s}(0)$. Use (for example) the MATLAB function `ode45` for the numerical integration.

Before going on to the calculations described below, make sure that the integration works as expected, e.g., that you get a (nearly) circular orbit if the initial vector is set to the motion of the LSR, i.e., $\boldsymbol{s}(0) = (-R_0, 0, 0, 0, V_0, 0)$ where R_0 is the distance of the sun from the Galactic Centre and V_0 the circular velocity at that distance.

Trajectories of a sample of stars in the solar neighbourhood

Stars in the solar neighbourhood (i.e., within a few hundred pc from the sun) can have rather different velocities. Let (U, V, W) denote the velocity of a star relative to the LSR. Its total velocity relative to the Galactic Centre (GC) is then $(U, V_0 + V, W)$. In P2 and P4 we estimated the velocity of the sun relative to the LSR, i.e., $(U_\odot, V_\odot, W_\odot)$. Use these values to compute the trajectory of the sun from its current position several orbital revolutions into the future. What is its maximum distance from the GC? Minimum distance? What is the period of the orbit around the GC? What is the radial oscillation period (the epicycle period)? What is the vertical oscillation period (in z)? What is the maximum height $|z|$ of the sun above or below the Galactic plane? How does the epicycle period compare to the value computed from the values of Oort's constants determined in P2?

The web pages for the course provide a file with the space velocities of 2086 stars within 100 pc from the sun. These velocities are expressed in km s^{-1} in a Galactic reference frame relative to the sun. That is, the table gives for each star the Hipparcos Catalogue number and $(U - U_\odot, V - V_\odot, W - W_\odot)$. Compute the trajectories for some of the stars. What are the radial and vertical amplitudes? What can be concluded concerning the birthplaces of these stars, compared to our sun?