P2: Oort's constants

Purpose

The purpose of this subproject is to estimate Oort's constants A and B from proper motion data in the Hipparcos Catalogue.

Theory

Oort's formulae for the galactic rotation are derived in Section 4 of the lecture notes. The relevant formula is that for the proper motion in galactic longitude, Eq. (4.13):

$$A \cos 2\ell + B = K\mu_\ell \tag{1}$$

where $K = 4.7405$ is the conversion factor from mas/yr to km/s/kpc and μ_ℓ is the proper motion in longitude. However, we have seen in P1 that the proper motions of nearby stars are dominated by the effect of the sun's motion through space, and also by the scatter of the individual velocities of the stars. The solar velocity is of order 20 km/s, and so is the typical random velocity of most other stars in the solar neighbourhood. It should be noted that the proper motion caused by the solar motion and random stellar velocities statistically *decreases* with distance from the sun, while the μ_ℓ predicted by (1) is *independent* of distance. This means that the former effects will dominate for nearby stars, while the latter dominates for distant stars.

To get a good result from (1) we should therefore select distant stars. However, even for them the solar motion could distort the result a little, and for this reason we may want to correct the proper motion for the (known) solar motion before using (1). If $\boldsymbol{v}_s$ is the solar motion determined in P1, the improved version of (1) is:

$$A \cos 2\ell + B = K\mu_\ell + (\boldsymbol{l} \cdot \boldsymbol{v}_s)p \tag{2}$$

where $\boldsymbol{l}$ is the unit vector in the direction of increasing ℓ and p is the parallax. The last term can be derived from Eq. (2.11) in the Lecture Notes and effectively corrects the observed μ_ℓ for the solar motion, so that it refers to the Local Standard of Rest (LSR) rather than the sun.

Oort's constants A and B are related to the rotation curve $V(R)$ of the Galaxy through:

$$V_0/R_0 = A - B\,, \quad (\mathrm{d}V/\mathrm{d}R)_0 = -(A + B) \tag{3}$$

where index 0 stands for quantities at the sun's distance from the galactic centre. Thus A and B give information about the rotation curve within a few kpc from the sun, where $V(R)$ is approximately linear.

Selection of stars

Use `Hipgal` to select distant stars of suitable colour at low galactic latitudes (small $|b|$). Save the astrometric data in a file which can be used as input for subsequent calculations.

Points to consider (and to answer in the report):

- Where does galactic rotation become important relative to the star's peculiar motions, or the solar motion?
- What can be considered 'distant' in this context?
- What can be considered 'small $|b|$' in this context?
- What might be a suitable colour interval in this case?

Calculations

Based on Eq. (1), use MATLAB to plot μ_ℓ versus ℓ and determine A and B. One possible method is to make a linear regression of μ_ℓ versus $\cos 2\ell$.

Then do the same thing but based on Eq. (2) instead. Any visible difference? Conclusions from this?

What are your best estimate of A and B?

Can you suggest (or even better, apply) a method to estimate the uncertainty of these values?

Report

The report should describe the purpose, data (including precise selection criteria), method, results, a discussion of (1) versus (2), and conclusions (e.g. comparison with textbook values).