P0: Preparatory Exercises

Purpose

To learn the basic use of MATLAB and Hipgal in the context of some problems in dynamical astronomy.

Exercise 1: The mass of the Coma cluster

Theory

The total mass M of an isolated n-body system in dynamical equilibrium can be estimated by means of the virial theorem as

$$M \simeq \frac{3\pi}{G}\,\sigma_{vr}^2\,d \tag{1}$$

where G is the gravitational constant and σ_{vr} is the dispersion of the radial velocities (v_r) of the galaxies. d is the harmonic mean of the projected separation of any pair of galaxies:

$$d = \left(\frac{1}{n(n-1)} \sum_i \sum_{j \neq i} \left[(x_i - x_j)^2 + (y_i - y_j)^2 \right]^{-1/2} \right)^{-1} \tag{2}$$

where (x_i, y_i) $(i = 1 \ldots n)$ is the projected position of the ith galaxy. The velocity dispersion is computed from the individual radial velocities v_{ri} as

$$\sigma_{vr} = \left(\frac{1}{n-1} \sum_i (v_{ri} - \bar{v}_r)^2 \right)^{1/2} \tag{3}$$

where $\bar{v}_r$ is the mean value of the radial velocities.

Data

The Coma cluster[1] contains over 1000 identified galaxies spread over several degrees on the sky. The distance to the centre of the cluster is about 94 Mpc. A list of 281 galaxies in the cluster can be found at `www.astro.lu.se/~lennart/Astrometry/` (follow the link under the section *Miscellaneous*). The file contains one line of data per galaxy. The first few lines of data look like this:

[1] See `en.wikipedia.org/wiki/Coma_galaxy_cluster` for general information about the cluster. For a picture, see `antwrp.gsfc.nasa.gov/apod/ap060321.html`

191.2250	27.7333	-163.04	-29.80	15.2	6539
191.2750	27.2667	-161.07	-57.80	14.7	7124
191.3500	27.5000	-156.73	-43.80	14.0	7917

Explanation to the columns:

Column 1: Right ascension (α_i) of the galaxy, in [deg]
Column 2: Declination (δ_i) of the galaxy, in [deg]
Column 3: Projected rectangular position (x_i) of the galaxy, in [arcmin]
Column 4: Projected rectangular position (y_i) of the galaxy, in [arcmin]
Column 5: Magnitude (m_i) of the galaxy, in [mag]
Column 6: Radial velocity (v_{ri}) of the galaxy, in [km s^{-1}]

where i is the index running from 1 to $n = 281$.

Calculations

On your PC, create a folder for the project. Open the web page with data for the Coma cluster, and save it as a text file in your folder. (Be careful not to save it as an html file. Check the file by means of a Notepad or WordPad to make sure that it only contains data.) In the following we assume that the data file is called `coma.txt`.

Start MATLAB. Change the Current Directory to your folder. Open the M-File Editor (File → New... → M-File) and type in the following script:

```
% script file for P0:1 (beginning)

% close all figures and clear variables from previous runs
close all;
clear;

% define some constants
G = 0.004301;   % gravitational constant in pc*(km/s)^2/Ms
distance = 93e6; % distance to the cluster in pc

% read from the file into the array data(:,:)
data = dlmread('coma.txt');

% find out how many rows there are (= number of galaxies)
n = size(data,1);
fprintf('\nnumber of galaxies, n = %d\n',n);
```

At this point you should save the M-script e.g. as `coma.m` (File → Save As...). Be sure to put the file in your own folder. Then run the script (e.g. using the button ). The result should be:

```
number of galaxies, n = 281
```

If you are not used to MATLAB, you should carefully go through every line of code so far (and below), making sure that you understand it completely. Use the Help function in MATLAB to find out what the non-trivial commands mean. Then you can continue coding the problem, e.g. making some plots. Add the following code in `coma.m`:

```
% store data in more convenient (one-dimensional) arrays
x = data(:,3);
y = data(:,4);
vr = data(:,6);

% plot the distribution of galaxies on the sky
figure(1);
plot(x, y, '+');
title('Coma cluster: Distribution of galaxies on the sky');
xlabel('x coordinate [arcmin]');
ylabel('y coordinate [arcmin]');
grid on;
```

Save and run the script again. This should produce a diagram such as in Fig. 1.

```
% plot their radial velocities vs. distance from cluster centre
figure(2);
hold on;
for i = 1:n
    r = sqrt(x(i)^2 + y(i)^2);
    plot(r, vr(i), 'or');
end
title('Coma cluster: Distribution of radial velocity versus radius');
xlabel('Projected distance from centre [arcmin]');
ylabel('Radial velocity [km/s]');
grid off;
```

This should give a plot like in Fig. 2. Actually, this plot can be made without the `for`-loop above, i.e. replacing the lines `hold on` ... `end` with a single plot command. How?

Finally, go on to make some serious calculations:

```
% calculate the harmonic mean of projected separations, Eq. (2)
sum = 0;
for i = 1:n
    for j = 1:n
        if (i ~= j)
            rij = sqrt( (x(i)-x(j))^2 + (y(i)-y(j))^2 );
            sum = sum + 1/rij;
        end
    end
end
d = n*(n-1)/sum;
d = distance * sin( pi * d/(180*60) );    % convert from arcmin to pc
fprintf('\nd = %d pc\n',d);
```

This calculates the harmonic mean projected separation and should give the result $d = 8.513282 \times 10^5$ pc. A few more lines of code are needed to get the virial mass. What is the result (in solar masses)?

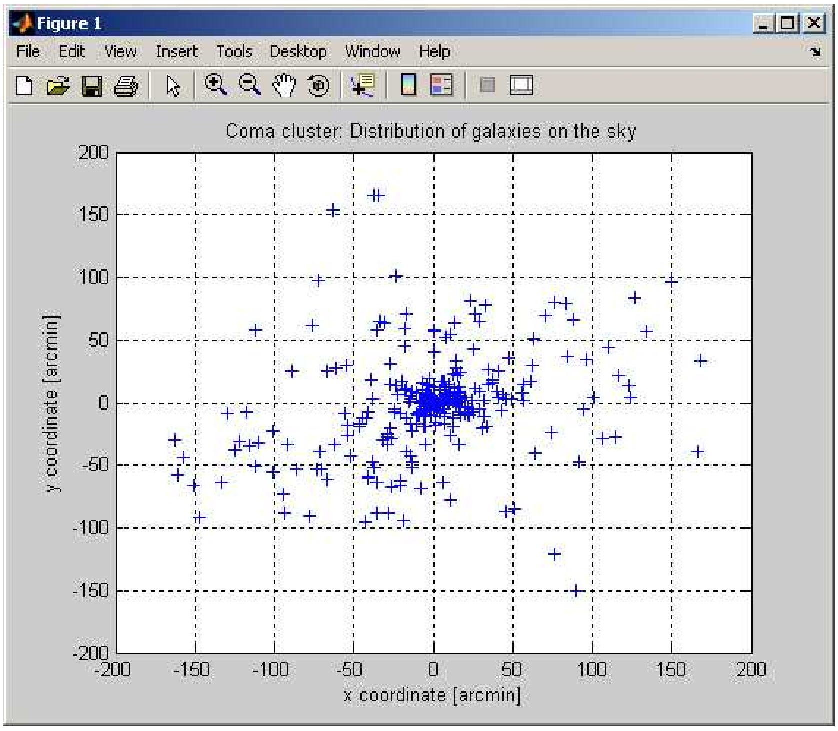

FIGURE 1: The MATLAB window in Exercise 1, showing a plot of the distribution of the galaxies on the sky.

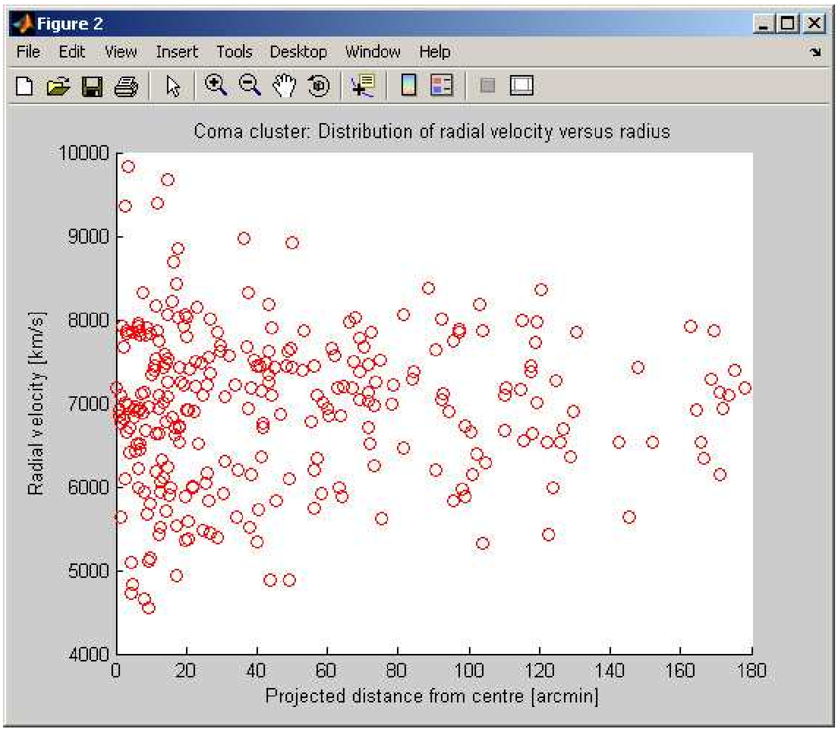

FIGURE 2: The MATLAB window in Exercise 1, showing a plot of the radial velocities versus distance from the centre of the cluster.

Exercise 2: The space distribution of bright stars

We use Hipgal to to study the space distribution of bright stars ($V < 7$). If the stars are uniformly distributed in space, one should see about the same number of stars per steradian for a given distance interval, independent of direction. Here we compare two regions on the sky:

1. A band of width 12.8 degrees along the galactic equator ($|b| < 6.4°$)
2. The union of two areas around the galactic poles ($|b| > 62.7°$)

The solid angle occupied by each region is about 1.4 sr. We also compare stars that are close to the sun (within 20 pc, i.e. $b > 50$ mas) with stars that are further away than 200 pc ($p < 5$ mas). Use Hipgal to find the number of stars in each case, and fill in the following table:

	$p > 50$ mas AND $V < 7$ mag	$p < 5$ mas AND $V < 7$ mag
$\|b\| < 6.4$ deg		
$\|b\| > 62.7$ deg		

Note that the conditions on $|b|$ can be programmed as follows:

- $|b| < 6.4$ is equivalent to $b > -6.4$ AND $b < 6.4$;
- $|b| > 62.7$ is equivalent to $b < -62.7$ OR $b > 62.7$.

Things to think about:

1. What conclusions can be drawn from the numbers in the above table?
2. Derive a formula for the solid angle of the sky between two arbitrary latitude circles, b_1 and b_2. Then calculate more precisely the solid angles of the two regions used above.

Exercise 3: The HR diagram

Use Hipgal and MATLAB to produce an HR diagram resembling Fig. 4 in the Lecture Notes.

Tip: to get only single stars with a relative distance uncertainty less than 10%, one can use the following criteria: multiplicity flag $= 0$, parallax error < 1.5 mas, parallax > 15 mas. How many such stars are there? Why are there many more stars in Fig. 4?

Hipgal does not allow to select stars by their relative distance uncertainty. This, and much more complex selection criteria, can of course be programmed in MATLAB. Try to make an HR diagram for single stars with $p > 10\sigma_p$.

Comment on the resulting HR diagram. What makes it different from the one in Fig. 4?

Reporting

No written reports need to be handed in for these exercises. However, you should be prepared to give and discuss the results during a subsequent lecture.