P4: The local mass density

Purpose

The purpose of this project is to calculate a dynamical estimate of the local mass density ρ_0 (in M_S pc^{-3}) based on the motions and number density profiles of tracer populations in the solar neighbourhood.

Theory

See the Lecure Notes (LN), in particular:

- Chapter 2 for calculating the heliocentric positions of the stars;
- Chapter 3 for the selection of stars to guarantee volume completeness;
- Chapters 9 and 10 (especially Sect. 10.1) for deriving the mass density.

In the plane parallel approximation (Sect. 10.1), the vertical component of Jeans' equation gives the following estimate of the mass density $\rho_0 = \rho(z \simeq 0)$ in the solar neighbourhood:

$$\rho_0 = -\frac{\sigma_w^2}{4\pi G}\left[\frac{\partial^2 \ln n(z)}{\partial z^2}\right]_{z=0} . \quad (1)$$

$n(z)$ is the number density of some tracer population as function of height (z) above the galactic plane, and σ_w is the velocity dispersion along the z axis of the same tracer population.

Different tracer populations (as distinguished, e.g., by their colours) have different velocity dispersions and different number density profiles, but they should give consistent estimates of the mass density.

Selection of stars

For this project we need tracer populations for which the number density $n(z)$ falls off significantly over the range of z values covered by the Hipparcos Catalogue. Moreover, since we must estimate the curvature of $n(z)$ over this range, it is extremely important that we really have the complete sample of tracer stars; otherwise we may observe an apparent reduction of $n(z)$ for large $|z|$ which merely reflects the growing incompleteness of the catalogue.

Since σ_w is needed, and may be taken from P2, it is a good idea to use the same intervals in $B-V$ as was used in P2. For each interval, calculate the distance $r_{\max}$ within which the Hipparcos Catalogue is complete, and study the variation of $n(z)$ for $-r_{\max} \le z \le r_{\max}$.

Method

The velocity dispersion σ_w is estimated as in P2, or taken directly from that project if the colour selection is the same. The main difficulty is to estimate the curvature $Q = \partial^2 \ln n / \partial z^2$ in Eq. (1). The proposed method is to divide the sphere $r \le r_{\max}$ (where $r_{\max} = 1000/p_{\min}$) into horizontal layers of constant thickness, e.g., $\Delta z = 10$ pc, count the number of stars in each layer and divide by the volume[1] to get $n(z)$ at the mean z coordinate of the layer. Then plot $\ln n(z)$ as a function of z and fit a parabola,

$$\ln n(z) \simeq c_1 z^2 + c_2 z + c_3 \tag{2}$$

e.g., using MATLAB's `polyfit` function. Then $Q = 2c_1$. Combining σ_w and Q gives the mass density estimate according to Eq. (1).

Repeat the calculation for different colour intervals. Only some of the colour intervals give a sensible result; for others the variation of $n(z)$ within the studied range of z values is simply too small, or too uncertain.

Discuss the significance of the different mass estimates and why they differ. Which colour interval works best? Why? What is your best estimate of the mass density? How uncertain is the result? Is the location of the maximum of Eq. (2) consistent with our picture of where the sun is located relative to the galactic plane? Is there any reason to suspect that the method systematically under- or overestimates the mass density?

Report

The report should describe the purpose, data (including precise selection criteria), method, results, plots, and a discussion of the results guided by the questions above. It should include a complete printout of the MATLAB code. (Other programming languages can be used instead of MATLAB.)

[1] The layer bounded by $r \le r_{\max}$ and $z_1 \le z \le z_2$ has volume $\pi r_{\max}^2 (z_2 - z_1) - \pi (z_2^3 - z_1^3)/3$ (provided that $|z_1| \le r_{\max}$ and $|z_2| \le r_{\max}$).