Exam Questions for ASTM13
Dynamical Astronomy

13–23 December 2011

Instructions:

Solve the following problems concisely, with reference to the lecture notes or other literature where appropriate.

The solutions should be sent by e-mail to mbd@astro.lu.se (single pdf file, no word documents), or handed in to me on paper (it I am not available, please put it in a sealed envelope in my mail box).

In either case, your solutions should be handed in before the end of Friday 23 December 2011.

Problems:

1. When observing Galactic stars in the direction $l \simeq 90$ degrees, one finds a number of stars with large negative radial velocities (for example, < -100 km/s), but extremely few with large positive radial velocities (for example, $> +100$ km/s). Explain this asymmetry.

2. A galaxy has a spherically-symmetric mass distribution with potential

$$\psi(\mathbf{r}) = -\frac{Gm}{\sqrt{r^2 + a^2}} \tag{1}$$

where $r = |\mathbf{r}|$ is the distance from the galactic centre, and m and a are positive constants. Derive: (a) the density profile $\rho(r)$; and (b) the mass within radius r, $M(r)$. Your answers should include complete derivations using formulae from the Lecture Notes, rather than just the resulting functions.

(c) What is the total mass of the galaxy? What radius encloses half the mass?

3. The collisionless Boltzmann equation (CBE) expresses the continuity of the 'stellar gas" in 6-dimensional phase space. This equation is only valid for a stellar gas that can be considered collisionless. Explain: (a) what is meant by collisionless in this context; (b) why this assumption is necessary for the CBE.

(c) M13 is a massive globular stellar cluster containing about three hundred thousand very old, low-mass stars. The stars have an internal velocity dispersion of about 10 km/s and a number density of about 10^4 stars/pc^3 in the centre of the cluster. Determine, using calculations as appropriate, whether the centre of M13 is collisionless.

4. Consider the potential in Equation (1). By deriving the relevant formulae, calculate: (a) the escape velocity for a particle as a function of distance, $V_{\rm esc}(r)$; (b) the speed of a particle on a circular orbit as a function of distance, $V_{\rm c}(r)$; and (c) the ratio $V_{\rm esc}(r)/V_{\rm c}(r)$. Comment on why this ratio changes with radius.

(d) A particle is orbiting within a galaxy with the potential given in Equation (1). At some time, it has a distance $r = a$, a tangential velocity $V_\phi = V_{\rm c}(r = a)/2$, and zero radial velocity. Describe the subsequent trajectory of this particle.