Exam questions for ASTM13

Dynamical astronomy

4–10 January 2010

Instructions:

Solve the following problems concisely, with reference to the lecture notes or other literature where appropriate.

The solutions should be sent by e-mail to lennart@astro.lu.se, or handed in to me on paper (if I am not available, please put it in a sealed envelope in my mail box).

In either case I want your solutions before the end of Sunday 10 January.

Problems:

1. When observing Galactic stars in the direction $\ell \simeq 90^\circ$, one finds a fair number of stars with large negative radial velocities (e.g., < -100 km s^{-1}), but extremely few with large positive radial velocities (e.g., $> +100$ km s^{-1}). Explain this asymmetry!

2. Assume that we have a spherically-symmetric mass distribution such that the acceleration of a test particle at the arbitrary point $\boldsymbol{r}$ is given by

$$\boldsymbol{a}(\boldsymbol{r}) = -\frac{Gm}{(r^2 + h^2)^{3/2}}\,\boldsymbol{r}\,. \qquad (1)$$

   The origin of $\boldsymbol{r}$ is at the centre of the mass distribution. m and h are fixed parameters of dimension mass and length, respectively. Using Poisson's equation, derive the radial density profile $\rho(r)$. What is the value of the central density, $\rho(0)$, in terms of m and h?

3. Consider a galaxy whose potential is given by the MiyamotoNagai potential, Eq. (7.1) in the lecture notes, with parameters a, b and M. For a star moving approximately in the symmetry plane ($z = 0$), in a nearly circular orbit at distance R from the centre, derive expressions for (a) the period of the circular orbit, and (b) the oscillation period for small excursions along the the z axis. (c) If $a = 0$ it turns out that the two periods are equal. Explain how this can be understood in terms of the geometries of the potential and of the orbit.

4. Consider the motions of two particles in a time-independent potential $\psi(x, y, z)$. Their paths through the phase space can be described by the two continuous functions $\boldsymbol{s}_1(t)$ and $\boldsymbol{s}_2(t)$, where $\boldsymbol{s} = (x, y, z, u, v, w)$ is the 6-dimensional phase space coordinate. At time $t = 0$ the two particles are at different points in the phase space. Explain why it is not possible for the two paths to cross, i.e., there cannot be two times t_1 and t_2 such that $\boldsymbol{s}_1(t_1) = \boldsymbol{s}_2(t_2)$.