Fundamental physics with Gaia

S.A. Klioner

Lohrmann-Observatorium, Technische Universität Dresden

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Why relativity?

Accuracy of astrometric observations

1 μas is the thickness of a sheet of paper seen from the other side of the Earth
Some relativistic effects in the data

Several general-relativistic effects are seen in the data with the following precisions:

- VLBI ± 0.0003
- HIPPARCOS ± 0.003
- Viking radar ranging ± 0.002
- Cassini radar ranging ± 0.000023
- Planetary radar ranging ± 0.0001
- Lunar laser ranging I ± 0.0005
- Lunar laser ranging II ± 0.007

Why general relativity?

- Newtonian models cannot describe high-accuracy observations:
  - many relativistic effects are many orders of magnitude larger than the observational accuracy

  ➞ space astrometry missions like Gaia would not work without relativistic modelling

- The simplest theory which successfully describes all available observational data:

  GENERAL RELATIVITY
Experimental foundations of General Relativity

Newtonian gravity

Based on physical ideas of Galileo Galilei and empirical findings of Johannes Kepler, Isaac Newton has provided a clear mathematical model of gravity:

$$m_A \ddot{x}_A = - \sum_{B \neq A} \frac{G m_A m_B}{|x_A - x_B|^3} (x_A - x_B)$$

Until 1859 the model explained all experimental facts within their observational accuracy.
Triumph of Newtonian gravity

Having performed analytical computations of incredible complexity, Urbain Leverrier in 1846 has predicted the position of a new planet Neptune:

Neptune was observed close to the predicted position on 23.09.1846 by Johann Gottfried Galle in Potsdam.

John Couch Adams has also predicted the position of a new planet, but the prediction was significantly less accurate than that of Leverrier and the observers in Greenwich were not successful.

M. Adams découvrant la nouvelle planète dans le rapport de M. Leverrier.
Assumptions of Newtonian gravity

The assumptions of Newtonian gravity can be read off the main equation:

\[ m_A^{\text{in}} \ddot{x}_A = - \sum_{B \neq A} \frac{G m_A^{\text{gr}} m_B^{\text{gr}}}{\left| \mathbf{x}_A - \mathbf{x}_B \right|^3} \left( \mathbf{x}_A - \mathbf{x}_B \right) \]

These are:

1) \( m^{\text{in}} = m^{\text{gr}} \) Weak equivalence principle (WEP)
2) \( G \neq G(t) \) \( G \) is constant both in time and in space
3) \( G \neq G(r) \)

Weak equivalence principle

inertial mass is equal (or proportional) to gravity mass

OR

all test bodies fall with the same acceleration

(Universality of Free Fall: Einstein’s elevator)

- The WEP was first tested by Galileo Galilei by throwing things from the Pisa tower: 0.02
Dropping a feather and hammer on the moon

Apollo 15, David Scott, 2.8.1971: live demonstration of the equivalence principle:

a heavy object (a 1.32-kg aluminum geological hammer) and a light object (a 0.03-kg falcon feather) were released simultaneously from approximately the same height (about 1.6 m) and reached the surface simultaneously.

„Mr. Galileo was correct!“

Weak equivalence principle: pendulum

different materials – equal periods

Galileo Galilei  (1590-1638)  0.02
Isaac Newton  (1680)  0.001
Friedrich Bessel (1830)  0.000017

Friedrich Wilhelm Bessel (1784-1846)
Weak equivalence principle: torsion pendulum

Loránd Eötvös (1848-1919)

detecting a torque on a hanging pendulum

Eötvös (1909) $5 \times 10^{-9}$
Braginsky-Panov (1972) $10^{-12}$
Adelberger (2003) $5 \times 10^{-13}$

Weak equivalence principle: free fall

- freely falling Earth and Moon: LLR $1.4 \times 10^{-13}$
- freely falling test bodies on an orbit around the Earth: Microscope, GG, STEP

Microscope: $10^{-15}$
STEP: $10^{-17}$
Weak equivalence principle

Relative difference between accelerations of two different bodies

Funded projects:

APOLLO (LLR): 10^{-14} @ 2015
MicroSCOPE: 10^{-15} @ 2012

Most ambitious unfunded idea:

STEP: 10^{-17}

Constancy of $G$ in space

Various physical ideas related to the search of new kinds of interactions lead to a modified law of gravity with

$$G(r) = G \left(1 + \alpha \left(1 + \frac{r}{\lambda}\right) \exp\left(-\frac{r}{\lambda}\right)\right)$$

Fifth force (1986-1995): $\lambda \sim 100$ m

$$G(r) \approx G r^{-n}, \quad n = 1, 2, \ldots$$

Some ideas in the string theory: $\lambda < 1$ mm

No deviations were found between $10^{-5}$ m to $10^{13}$ m
**Constancy of $G$ in space**

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---

**Constancy of $G$ in time**

If $G$ were time-dependent, the motion of planets would have a specific behaviour in time: linear drift of the periods of motion.

This can be tested in the solar system!

$$\frac{\dot{G}}{G}, \quad \text{yr}^{-1}$$

- Moon: $<7 \cdot 10^{-13}$
- Planets: $<5 \cdot 10^{-13}$
- Asteroids: $<10^{-10}$

many independent groups confirm these results...

**Funded projects:**

**APOLLO (LLR):** $10^{-14}$ @ 2015

**Warning:**

masses become time-dependent below $10^{-13}$ / yr!
The first experimental fact contradicting Newtonian theory of gravity

The perihelion advance of Mercury discovered 1859 by Leverrier

How to explain the perihelion advance?

Many ideas were proposed to explain the anomalous perihelion advance of Mercury:

A) Additional bodies:
   - additional planet between Mercury and Sun (Vulcan)
   - rings of dust or minor bodies of very special forms and masses

B) Various modifications of the Newtonian attraction law
   - \( F \sim 1/r^2 + \epsilon \)
   - \( F = F(r,v) \)
   - ...

All failed!

The problem was to find an explanation for the perihelion advance of Mercury, which does not destroy other predictions (e.g. motion of the Moon) of Newtonian gravity
... and the answer was

**General Relativity Theory**

---

**Einstein equivalence principle**

The Einstein Equivalence Principle (EEP) consists of 3 parts:

1. **Weak Equivalence Principle (WEP)**: no matter what bodies we observe
2. **Local Lorentz Invariance (LLI)**: no matter how we move
   - the outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed.
3. **Local Positional Invariance (LPI)**: no matter where and when
   - the outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed.

Free fall or at rest far away from all masses?

Accelerated (an elevator with thrusters) or at rest in a homogeneous gravitational field?

No way to decide!
Local Lorentz Invariance

- Motivated by ideas about quantum gravity, a tremendous amount of effort over the past decade has gone into testing Lorentz invariance in various regimes.

- Details: David Mattingly, Living Rev. Relativity, 8, (2005), 5

- Simplest approach: Robertson, 1948; Mansouri, Sexl, 1977:

  preferred frame 1: \((T, X^i)\)
  
  light velocity is constant: \(c^2 dT^2 = dX^2 + dY^2 + dZ^2\)

  frame 2: \((t, x^i)\)
  
  light velocity is no longer constant…

  e.g. frame 1 could be the frame where the Cosmic Microwave Background looks isotropic: \(v_\circ \approx 370\) km/s

  \[ \alpha = 11.2^h, \delta = -6.4^\circ \]

Local Lorentz Invariance

- Transformation between these two frames:

  \[
  dT = \frac{1}{a} (dt + \frac{v^2}{c^2} dx) \quad a = 1 + \alpha \frac{v^2}{c^2} + \mathcal{O}(c^{-4})
  \]

  \[
  dX = \frac{1}{b} dx + \frac{1}{a} (dt + \frac{v^2}{c^2} dx) \quad b = 1 + \beta \frac{v^2}{c^2} + \mathcal{O}(c^{-4})
  \]

  \[
  dY = \frac{1}{c} dy \quad d = 1 + \delta \frac{v^2}{c^2} + \mathcal{O}(c^{-4})
  \]

  Special Relativity: \(\alpha = -1/2, \beta = 1/2, \delta = 0\)

- Light velocity in frame 2:

  \[
  dt = \frac{dl}{c} \left[ 1 - (\beta - \alpha - 1) \frac{v^2}{c^2} - \left(\frac{1}{2} - \beta + \delta\right) \sin^2 \theta \frac{v^2}{c^2} \right] + \mathcal{O}(c^{-4})
  \]
Local Lorentz Invariance

- Three classic experiments:
  \[
  P_{MM} = \frac{1}{2} - \beta + \delta \\
  P_{KT} = \beta - \alpha - 1 \\
  P_{IS} = |\alpha + 1/2|
  \]
  Michelson-Morley: orientation dependence
  Kennedy-Thorndike: velocity dependence
  Ives-Stillwell: contraction, dilation

\[
P_{MM} = 9.4 (\pm 8.1) \times 10^{-11}
\]
Stanwix et al, PRD 74 (2006) 081101

\[
P_{KT} = -3.1 (\pm 6.9) \times 10^{-7}
\]
Wolf et al, PRL 90 (2003) 060402

\[
P_{IS} < 2.2 \times 10^{-7}
\]
Saathoff et al, PRL 91 (2003) 190403

Local Lorentz Invariance

The degree of the violation of Lorentz Invariance in electromagnetism

\[
\delta = \frac{c_0^2}{c^2} - 1
\]
Will, 2005
Local Positional Invariance

One aspect of the LPI can be tested by measuring the gravitational red shift of clocks

degree of the violation of the gravitational red shift

\[ \frac{\Delta \nu}{\nu} = \left(1 + \alpha\right) \frac{\Delta U}{c^2} \]

Metric theories of gravity

- If the Einstein Equivalence Principle is valid, gravitation must be a phenomenon of curved space-time described by a **metric theory of gravity.**

- A theory of gravity is called metric theory of gravity if:
  - space-time is endowed with a symmetric metric
  - the trajectories of freely falling test bodies are geodesics of that metric
  - in local freely falling reference frames, the non-gravitational laws of physics are those written in the language of special relativity

- General Relativity is the simplest metric theory of gravity

- There are very many others
Parametrized post-Newtonian (PPN) formalism

- K. Nordtvedt, C. Will (1970-)

- covers a class of possible metric theories of gravity in the weak-field slow-motion (post-Newtonian) approximation:

  many metric theories of gravity were investigated and a generic form of the post-Newtonian metric tensor of a system of N bodies was derived.

- the metric tensor contains 10 numerical ad hoc parameters.

- Two most important parameters are $\gamma$ and $\beta$ ($\gamma = \beta = 1$ in GRT)

- All predictions of the theories can be expressed using these parameters

General Relativity predicts the perihelion advance

Einstein’s General Theory of Relativity has naturally explained the perihelion advance of Mercury.

$$\Delta \omega = 2\pi \frac{(2\gamma + 2 - \beta)GM}{c^2 a (1 - e^2)}$$, rad per revolution

Modern precision of the perihelion shift: $\sim 10^{-3}$
Second test of General Relativity: light deflection

Light deflection from the Sun: 1.75"

\[ \Delta \varphi = \frac{2(1 + \gamma)GM}{c^2d} \]

Eddington's expedition measures the deflection during the total solar eclipse 29 May 1919: Sobral (Brazil), Principe (island close to Africa)

Conceivable outcomes:
- No deflection = 0
- Newton = 0.87"
- Einstein = 1.75"

Einstein and Eddington, Cambridge, 1930

one of Eddington's photographs of the 1919 eclipse, presented in:

Dyson, F.W., Eddington, A.S., & Davidson, C.R. 1920
Mem. R. Astron. Soc., 220, 291-333:

1.98” ± 0.12”
1.61” ± 0.30”
Third test of General Relativity: Shapiro delay

Light needs a bit longer to go from the emitter to the receiver than the distance between them divided by $c$.

$$t = \frac{1}{c} |\mathbf{x}_{\text{emitter}} - \mathbf{x}_{\text{receiver}}| + \frac{(1 + \gamma)GM}{c^3} F(\mathbf{x}_\odot, \mathbf{x}_{\text{emitter}}, \mathbf{x}_{\text{receiver}})$$

Discovered by Irwin Shapiro in 1964 as a theoretical prediction of General Relativity.

First measured by the Shapiro’s team at the end of the 1960s with an accuracy of 10%.

Light propagation: modern tests

[Diagram showing experimental results for the parameter $(1+\gamma)/2$ over time.]
Other tests of General Relativity

- Geodetic precession (with Lunar Laser Ranging) \( \pm 0.009 \)
- Gravitomagnetism (with Satellite Laser Ranging) \( \pm 0.1 \)
- Nordtvedt effect (with Lunar Laser Ranging) \( \pm 0.001 \)

reflectors on the Moon and on satellites

Testing in \( \beta-\gamma \) plane of the PPN formalism

![Diagram showing testing results in the \( \beta-\gamma \) plane.](image)
Strong field tests

- Binary pulsar PSR1913+16: indirect evidence for gravity waves
- Double pulsar PSR J0737-3039A/B: more precise
- Existence of black holes:
  - stellar mass (Cyg X1)
  - supermassive black hole in the centers of galaxies
    - IR measurements of the stellar orbits around the center of Milky Way

Observations against General Relativity?

- Are there some experimental evidences against General Relativity?

  “Candidates”:

  “Pioneer anomaly”: unexpected additional constant acceleration of two Pioneer spacecrafts directed towards the Sun $8.5 \times 10^{-9} \text{ m/s}^2$

  Criticism: “dirty” models of the spacecrafts (heat radiation from RTG, etc.)

  “Fly-by anomaly”: unexpected additional increase of the velocity of several spacecrafts of Earth fly-bys

  Criticism: the used model is a simplification of a consistent relativistic one; instrumental noise…

Could the story of the Le Verrier’s discovery of the perihelion advance repeat? A lot of care should be taken here!
The standard framework for relativistic modelling

Astronomical observation

- Observables are directly related to the inertial coordinates.
- Physically preferred global inertial coordinates.
Astronomical observation

no physically preferred coordinates

observables have to be computed as coordinate independent quantities

General relativity for space astrometry

Relativistic reference system(s)

- Relativistic equations of motion
- Equations of signal propagation
- Definition of observables

Relativistic models of observables

Coordinate-dependent parameters

Observational data
Metric tensor and reference systems

• In relativistic astrometry
  • BCRS (Barycentric Celestial Reference System)
  • GCRS (Geocentric Celestial Reference System)
  • Local reference system of an observer

play an important role.

• All these reference systems are defined by

  the form of the corresponding metric tensor.

Metric tensor

• Pythagorean theorem in 2-dimensional Euclidean space \( \mathbb{R}^2 \)

\[
(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2
\]

• length of a curve in \( \mathbb{R}^2 \)

\[
\ell = \int_A^B ds
\]

\[
ds^2 = dx^2 + dy^2 = dr^2 + r^2 d\theta^2 = \sum_{i=1}^{2} \sum_{j=1}^{2} g_{ij} dx^i dx^j
\]
Metric tensor: special relativity

- special relativity, inertial coordinates

\[ x^\mu \equiv (x^0, x^i) = (ct, x, y, z) \]

- The constancy of the velocity of light in inertial coordinates

\[ dx^2 = c^2 dt^2 \]

can be expressed as \( ds^2 = 0 \) where \( ds^2 = -c^2 dt^2 + dx^2 \)

\[
\begin{align*}
g_{00} &= -1, \\
g_{0i} &= 0, \\
g_{ij} &= \delta_{ij} = \text{diag}(1, 1, 1).
\end{align*}
\]

Barycentric Celestial Reference System

The BCRS:

- adopted by the International Astronomical Union (2000)
- suitable to model high-accuracy astronomical observations

\[
\begin{align*}
g_{00} &= -1 + \frac{2}{c^2} w(t, x) - \frac{2}{c^4} w^2(t, x), \\
g_{0i} &= -\frac{4}{c^3} w^i(t, x), \\
g_{ij} &= \delta_{ij} \left(1 + \frac{2}{c^2} w(t, x)\right).
\end{align*}
\]

\( w, w^i \): relativistic gravitational potentials
Barycentric Celestial Reference System

The BCRS is a particular reference system in the curved space-time of the Solar system.

- One can use any
- but one should fix one

Geocentric Celestial Reference System

The GCRS is adopted by the International Astronomical Union (2000) to model physical processes in the vicinity of the Earth:

A: The gravitational field of external bodies is represented only in the form of a relativistic tidal potential.
B: The internal gravitational field of the Earth coincides with the gravitational field of a corresponding isolated Earth.
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\[
G_{00} = -1 + \frac{2}{c^2} W(T, X) - \frac{2}{c^4} W^2(T, X),
\]
\[
G_{0a} = -\frac{4}{c^3} W^a(T, X),
\]
\[
G_{ab} = \delta_{ab} \left( 1 + \frac{2}{c^2} W(T, X) \right).
\]

\( W, W^a \): internal + inertial + tidal external potentials
Why not the BCRS?

- Imagine a sphere (in inertial coordinates of special relativity), which is then forced to move in a circular orbit around some point...

- What will be the form of the sphere for an observer at rest relative to that point?

Lorentz contraction deforms the shape...

Additional effect due to acceleration (not a pure boost) gravity (general relativity, not special one)

Local reference system of an observer

The version of the GCRS for a massless observer:

A: The gravitational field of external bodies is represented only in the form of a relativistic tidal potential.

$W, W^a : \text{internal + inertial + tidal external potentials}$

- Modelling of any local phenomena:
  observation, attitude, local physics (if necessary)
Equations of translational motion

- The equations of translational motion (e.g. of a satellite) in the BCRS

\[ g_{00} = -1 + \frac{2}{c^2} w(t, x) - \frac{2}{c^2} w^2(t, x), \]
\[ g_{0i} = -\frac{4}{c^3} w'(t, x), \]
\[ g_{ij} = \delta_{ij} \left( 1 + \frac{2}{c^2} w(t, x) \right). \]

- The equations coincide with the well-known Einstein-Infeld-Hoffmann (EIH) equations in the corresponding limit

\[
\ddot{x}_A = -\sum_{B \neq A} G M_B \frac{x_A - x_B}{|x_A - x_B|^3} + \frac{1}{c^2} F(t)
\]

Equations of light propagation

- The equations of light propagation in the BCRS

\[ g_{00} = -1 + \frac{2}{c^2} w(t, x) - \frac{2}{c^4} w^2(t, x), \]
\[ g_{0i} = -\frac{4}{c^3} w'(t, x), \]
\[ g_{ij} = \delta_{ij} \left( 1 + \frac{2}{c^2} w(t, x) \right). \]

- Relativistic corrections to the “Newtonian” straight line:

\[ x(t) = x_0(t) + c \sigma(t - t_0) + \frac{1}{c^2} \Delta x(t) \]
Observables I: proper time

Proper time $\tau$ of an observer can be related to the BCRS coordinate time $t=TCB$ using

- the BCRS metric tensor
- the observer’s trajectory $x^i_o(t)$ in the BCRS

$$\frac{d\tau}{dt} = 1 + \frac{1}{c^2} A_{pN} + \frac{1}{c^4} A_{ppN}$$

$$g_{00} = -1 + \frac{2}{c^2} w(t,x) - \frac{2}{c^4} w^2(t,x),$$

$$g_{0i} = -\frac{4}{c^3} w'(t,x),$$

$$g_{ii} = \delta_{ii} \left( 1 + \frac{2}{c^2} w(t,x) \right).$$

Observables II: proper direction

- To describe observed directions (angles) one should introduce spatial reference vectors moving with the observer explicitly into the formalism

- Observed angles between incident light rays and a spatial reference vector can be computed with the metric of the local reference system of the observer
Data analysis models compatible with the IAU 2000

- Ephemeris construction (JPL, IMCCE, IAA) OK
- VLBI OK
- Lunar Laser Ranging partially OK ⇒ OK
- Satellite Laser Ranging OK
- Hipparcos, Gaia, … OK
- time keeping and time transfer algorithms OK
- pulsar timing OK
- GPS OK / not OK
- Earth/Moon rotation not OK ⇒ OK

All data kinds are “under control”…

Relativistic Model for Gaia
The standard astrometric model

- $s$ the observed direction
- $n$ tangential to the light ray at the moment of observation
- $\sigma$ tangential to the light ray at $t = -\infty$
- $k$ the coordinate direction from the source to the observer
- $l$ the coordinate direction from the barycentre to the source
- $\pi$ the parallax of the source in the BCRS

Sequences of transformations

- Stars:
  $s \leftrightarrow n \leftrightarrow \sigma \leftrightarrow k \leftrightarrow l(t), \pi(t) \leftrightarrow l_0, \pi_0, \mu_0, \dot{\pi}_0, \ldots$

- Solar system objects:
  $s \leftrightarrow n \leftrightarrow k \leftrightarrow$ orbit

(1) aberration
(2) gravitational deflection
(3) coupling to finite distance
(4) parallax
(5) proper motion, etc.
(6) orbit determination
Aberration: \( s \leftrightarrow n \)

- Lorentz transformation with the scaled velocity of the observer:

\[
s = \left(-n + \left\{ \frac{\gamma}{c} - (\gamma - 1) \frac{v \cdot n}{v^2} \right\} v \right) \frac{1}{\gamma (1 - v \cdot n / c)} ,
\]

\[
\gamma = \left(1 - v^2 / c^2 \right)^{-1/2} ,
\]

\[
v = \dot{x}_o \left(1 + \frac{2}{c^2} w(t, x_o) \right)
\]

- For an observer on the Earth or on a typical satellite:

  - Newtonian aberration
  - Relativistic aberration
  - Second-order relativistic aberration

  - Requirement for the accuracy of the orbit: \( |\delta s| \leq 1 \mu \text{as} \) \( \Rightarrow \) \( |\delta \dot{x}_o| \leq 1 \text{mm/s} \)

Gravitational light deflection: \( n \leftrightarrow \sigma \leftrightarrow k \)

- Several kinds of gravitational fields deflecting light
  - Monopole field
  - Quadrupole field
  - Gravitomagnetic field due to translational motion
  - [ Gravitomagnetic field due to rotational motion ]
  - [ Post-post-Newtonian corrections (ppN) ]
Gravitational light deflection: \( n \leftrightarrow \sigma \leftrightarrow k \)

- The principal effects due to the major bodies of the solar system in \( \mu \text{as} \)
- The maximal angular distance to the bodies where the effect is still \( > 1 \mu \text{as} \)

<table>
<thead>
<tr>
<th>body</th>
<th>Monopole</th>
<th>( \psi_{\text{max}} )</th>
<th>Quadrupole</th>
<th>( \psi_{\text{max}} )</th>
<th>ppN</th>
<th>( \psi_{\text{max}} )</th>
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<tbody>
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<td>Sun</td>
<td>( 1.75 \times 10^6 )</td>
<td>180°</td>
<td></td>
<td></td>
<td>11</td>
<td>53’</td>
</tr>
<tr>
<td>(Mercury)</td>
<td>83</td>
<td>9’</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Venus</td>
<td>493</td>
<td>4.5°</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Earth</td>
<td>574</td>
<td>125°</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Moon</td>
<td>26</td>
<td>5°</td>
<td></td>
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<tr>
<td>Mars</td>
<td>116</td>
<td>25’</td>
<td></td>
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<tr>
<td>Jupiter</td>
<td>16270</td>
<td>90°</td>
<td>240</td>
<td>152”</td>
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<td>17°</td>
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<td>46”</td>
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<td>Uranus</td>
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<td>71’</td>
<td>8</td>
<td>4”</td>
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<td>51’</td>
<td>10</td>
<td>3”</td>
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Gravitational light deflection: \( n \leftrightarrow \sigma \leftrightarrow k \)

- Monopole light deflection: distribution over the sky on 25.01.2006 at 16:45
  equatorial coordinates
Gravitational light deflection: $n \leftrightarrow \sigma \leftrightarrow k$

• Monopole light deflection: distribution over the sky on 25.01.2006 at 16:45 equatorial coordinates

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</tr>
<tr>
<td>Neptune</td>
<td>2533</td>
<td>51 ’</td>
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</table>

Animation by Jos de Bruijne
Gravitational light deflection: $n \leftrightarrow \sigma \leftrightarrow k$

- A body of mean density $\rho$ produces a light deflection not less than $\delta$ if its radius:

$$R \geq \left( \frac{\rho}{1 \text{ g/cm}^3} \right)^{-1/2} \times \left( \frac{\delta}{1 \mu\text{as}} \right)^{1/2} \times 650 \text{ km}$$

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<td>Umbriel</td>
<td>1</td>
</tr>
<tr>
<td>Ceres</td>
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Parallax and proper motion: $k \leftrightarrow l \leftrightarrow l_0, \mu_0, \pi_0$

- All formulas here are formally Euclidean:

$$k = \frac{x_o(t_o) - X_s(t_e)}{|x_o(t_o) - X_s(t_e)|}, \quad l = \frac{X_s(t_e)}{|X_s(t_e)|},$$

$$X_s(t_e) = X_s(t_{e0}) + V_s(t_{e0})(t_e - t_{e0}) + \ldots$$

- Expansion in powers of several small parameters:

$$\pi = \frac{1 \text{ AU}}{|X_s(t_e)|}, \quad \mu = \frac{|V_s(t_e)|}{|X_s(t_e)|}$$

$$k = -l + \ldots, \quad l = l_0 + \ldots$$
Celestial Reference Frame

- All astrometrical parameters of sources obtained from astrometric observations are defined in BCRS coordinates:
  - positions
  - proper motions
  - parallaxes
  - radial velocities
  - orbits of minor planets, etc.
  - orbits of binaries, etc.

- These parameters represent a realization (materialization) of the BCRS

- This materialization is „the goal of astrometry“ and is called

Beyond the standard model

- Gravitational light deflection caused by the gravitational fields generated outside of the solar system
  - microlensing on stars of the Galaxy,
  - gravitational waves from compact sources,
  - primordial (cosmological) gravitational waves,
  - binary companions, …

ONLY TIME-DEPENDENT EFFECTS ARE IMPORTANT!

Microlensing noise and stochastic gravitational wave background could be crucial for going well below 1 microarcsecond…
Necessary condition: consistency of the whole data processing chain

- Any kind of inconsistency is very dangerous for the quality and reliability of the resulting catalogue.

- The whole data processing and all the auxiliary information should be assured to be compatible with the PPN formalism (or at least GR)
  - planetary ephemeris: coordinates, scaling, constants
  - Gaia orbit: coordinates, scaling, constants
  - astronomical constants
  - ???

- Monitoring of the consistency during the whole project

---

Example 1: Lissajous orbit around L₂

Because of Newtonian aberration, the velocity must be known with an accuracy of 1 mm/s in $10^3$ km from L₂.
Relativistic description of the Gaia orbit

Relativistic effects for the Lissajous orbits around $L_2$ (Klioner, 2005)

Example: Differences between position for the Newtonian and relativistic dynamical models in km vs. time in days

Deviations grow exponentially:

\[
\begin{align*}
\text{Log}(dX \text{ in km}) & \quad \text{Log}(dV \text{ in mm/s}) \\
\text{Newton} – \text{none} & \quad \text{purely Newtonian force} \\
S & \quad \text{Sun} \\
S+E & \quad \text{Jupiter} \\
S+E+J & \quad \text{Earth} \\
S+E+M & \quad \text{Moon}
\end{align*}
\]
Example 2: Optical aberrations by a rotating instrument

- Two special-relativistic effects modifying PSF of a rotating instrument:
  - Finite light velocity leads to propagation delays within telescope; these delays depend on the position in the field of view
  - Special-relativistic change of the reflection law (Einstein, 1905)

- Reassessment study for Gaia was necessary

Optical aberrations by a rotating instrument

- Aberration patterns by the instrument at rest

Optical aberrations by a rotating instrument

- Aberration patterns by the rotating instrument (VERY large angular velocity!)
Testing Relativity with Gaia

Why to test further?

intentionally left blank
Why to test further?

alternative theories of gravity...

Relativity as a driving force for Gaia
Gaia’s main goals for testing relativity

Light deflection:

Perihelion and node precessions:

Secular change of $G$:

\[ \sigma_\gamma < 5 \cdot 10^{-7} \]
\[ \sigma_\beta < 10^{-3} \]
\[ \sigma_{J_2 \odot} < 10^{-7} \]
\[ \sigma_{\dot{G}/G} < 5 \times 10^{-13} \text{ yr}^{-1} \]

Improved ephemeris → Fundamental physics with Gaia → Consistency checks

Global tests

Local tests

Differential solutions

Monopole

Quadrupole

Gravimagnetic

Asteroids

Perihelion precession

Non-Schwarzschild effects

SEP with the Trojans

$J_2$ of the Sun

$\dot{G}/G$
The mean rate of the proper time on the Gaia orbit is different from Terrestrial Time (or TAI) by about $6.9 \times 10^{-10}$.

Periodic terms of order $1 - 2 \mu s$.

The gravity term itself is about $8 \times 10^{-10}$ (20 times larger than for the ISS).


The accuracy is still unclear since not all technical details are fixed by now…

Special-relativistic aberration is given by

$$ s = \left( -n + \left\{ \frac{\gamma}{c} - (\gamma - 1) \frac{v \cdot n}{v^2} \right\} v \right) \frac{1}{\gamma(1-v \cdot n/c)}, $$

$$ \gamma = \left( 1 - \frac{v^2}{c^2} \right)^{-1/2}, $$

$$ v = \dot{x}_o \left( 1 + \frac{2}{c^2} U(t, x_o) \right) $$

The validity of this formula and, therefore, of the Local Lorentz Invariance can be tested:

- first-order terms: $10^{-10} - 10^{-11}$
- second-order terms: $10^{-6} - 10^{-7}$
PPN $\gamma$ from light deflection

- Most precise test possible with Gaia

$$\sigma_\gamma < 5 \cdot 10^{-7}$$

- Advantages of the Gaia experiment
  - optical,
  - deflection (not a Shapiro effect),
  - wide range of angular distances,
  - full-scale simulations of the experiments

- Problems with some of the "current best estimates" of $\gamma$
  1. special fits of the post-fit residuals of a standard solution (e.g., missed correlations leads to wrong estimates of the uncertainty);
  2. no special simulations with faked data to check what kind of effects we are really sensitive to

---

**Gaia sensitivity to the gravitational light deflection due to the Sun**

135 @ 60°

![Graph showing Gaia sensitivity to light deflection due to the Sun](image-url)
Gaia sensitivity to the gravitational light deflection due to the Sun

wide range of angular distances: mapping out the light deflection

Light deflection from the planets

Jupiter: monopole

\[ \gamma: 1.1 \times 10^{-3} \]

gradient-gravitomagnetic

\[ \alpha: 2.5 \times 10^{-3} \]

quadrupole

\[ \varepsilon: >0.1 \]

For other planets the results are worse: 0.1-0.007 for the monopole

Problem: rings, dust, gas, etc. in the vicinity of the giant planets
Relativistic effects with asteroids

Perihelion precession due to the Sun:

historically the first test of general relativity

<table>
<thead>
<tr>
<th>Object</th>
<th>$\Delta \omega (&quot;/cty)$</th>
<th>$e \Delta \omega (&quot;/cty)$</th>
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<th>$e$</th>
<th>$i (^\circ)$</th>
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<td>1.52</td>
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Perihelion precession (the first 20001 asteroids)

<table>
<thead>
<tr>
<th>Object</th>
<th>number</th>
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<th>$e \Delta \omega (&quot;/cty)$</th>
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<th>$e$</th>
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## Perihelion precession (12.09.05: 253113)

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</tr>
</tbody>
</table>

## Relativistic effects with asteroids

Preliminary results: Hestroffer, Berthier, Mouret, Mignard, 2004-

\[ \sigma_\beta < 10^{-3} \]
\[ \sigma_{J_2} < 10^{-7} \]
\[ \sigma_{\dot{G}/G} < 5 \times 10^{-13} \text{ yr}^{-1} \]

Smaller relativistic effects in the motion of asteroids should still be investigated…
Relativistic effects with asteroids

Non-Schwarzschild effects: relativistic N-body problem (N>2)

- Orbital consequences of the Einstein-Infeld-Hoffman equations for asteroids are still poorly known.

- Especially interesting for resonant asteroids for which the relativistic effects of e.g. Jupiter can be enhanced

Maximal „post-Sun“ perturbations in meters

\[ \Delta_2 = |x_{N+Sun} - x_{pN}| \]

200,000 Integrations over 200 days

Preliminary results: some asteroids with small relativistic perihelion precession have large relativistic 3-body effects
Relativistic effects in asteroids

The Nordtvedt effect with Trojan asteroids (and some other resonant ones)

- Historically the first example of an observable effect due to a violation of the Strong Equivalence Principle (Nordtvedt, 1968):
  
  shift of L4 and L5 by 1" for $\eta=1$

- Orellana, Vucetich, 1993: $\eta=-0.54\pm0.48$
  12 Trojans,
  100-200 observations for each,
  accuracy 1"

- Gaia: $10^3$-$10^4$ better

Pattern matching in positions/proper motions

I. Acceleration of the Solar system relative to remote sources leads to a time dependency of secular aberration: $\sim5\,\mu$as/yr

- constraint for the galactic potential model
- important for the binary pulsar test of relativity (at 1% level)

Mathematics:

expansion of the proper motion field into vector spherical harmonics

$$\vec{\mu} = \sum_{n=0}^{\infty} \sum_{m=0}^{n} a_{nm}^E \vec{y}_{nm}^E + a_{nm}^M \vec{y}_{nm}^M$$

the coefficients for $n=1$ give
  - rotations
  - the solar system acceleration

1993- :Ongoing unsuccessful attempts with geodetic VLBI
Gaia will measure the acceleration with at least 10% accuracy
Pattern matching in positions/proper motions

II. Constraint on very low frequency gravitational waves:

- constraint of stochastic GW flux with $\omega < 3 \times 10^{-9}$ Hz
  (similar study done for VLBI: Pyne et al. 1996, 1997)

- attempts to fit a pattern of apparent motions induced by an individual GW with $\omega < 3 \times 10^{-8}$ Hz

Example: a GW of strain $h$ and frequency $\omega$ propagating in the direction $\delta=90^\circ$:

$$\hat{\mu} = \frac{1}{2} \omega h \sin \omega T \cos \delta \left( \cos 2\alpha \hat{e}_\delta + \sin 2\alpha \hat{e}_\alpha \right)$$

The harmonic coefficients for $n>1$ give the GW-flux constraints

From Gaia for $\omega < 3 \times 10^{-9}$ Hz:

$$\Omega_{GW} < (0.001 \div 0.005) h^{-2}$$

Individual relativistically interesting objects: can Gaia provide a test for the existence of black holes?

- Fuchs, Bastian, 2004: Weighing stellar-mass black holes in binaries
- Astrometric wobble of the companions (just from binary motion)

<table>
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<td>XTEJ1550-564</td>
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</tbody>
</table>

- Already known objects:
- Unknown objects, e.g. binaries with “failed supernovae” (Gould, Salim, 2002)
- Gaia advantage: we record all what we see!
Summary

One sentence from each part

• Gaia would not work without relativistic modelling

• General Relativity is a well-established physical theory with many application at the engineering level

• The standard framework for relativistic modelling is used for virtually all kinds of high-accuracy astrometric observations

• Relativistic model for Gaia is conceptually simple, the main challenge being the consistency of the whole data processing chain

• Gaia will provide a variety of relativistic tests