Binary Stars

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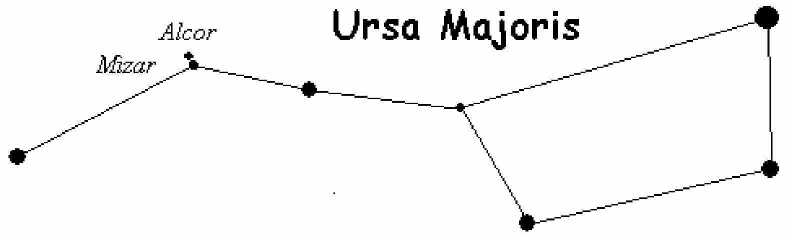
Summary

1. The "double star zoo": visual binaries (wide and orbital), astrometric b., spectroscopic b., eclipsing b.. General description and physical parameters.
2. Statistical properties of unevolved binaries: derivation and interpretation.
3. Formation models.

Wide Visual Binaries (1)



Mizar and Alcor : a 1800 AU binary ?

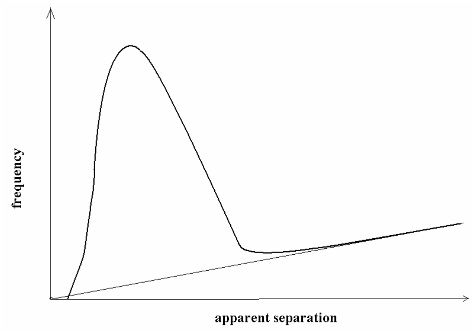
1768 : Michell calculates the probability to get optical double stars by random, and concludes it is too low for considering that double stars are just due to perspective.

1852: Struve's formula (1852) : When N stars are in the area A, the number of optical pairs closer than separation ρ is:

$$n(\rho) = N(N-1)\,\pi\,\rho^2/2A$$

Wide Visual Binaries (2)

The distribution of apparent separations: a linear function for optical pairs. At the opposite, the physical binaries are as close as permitted by the resolution of the instrument.



This technique for discriminating optical and physical binaries is still more efficient when applied to pairs of stars with common parallaxes, proper motions and radial velocities.

2007 : 102 492 systems in the " Washington Double Star " catalog, ***How much after Gaia ?***

The 2-body problem: Kepler's laws

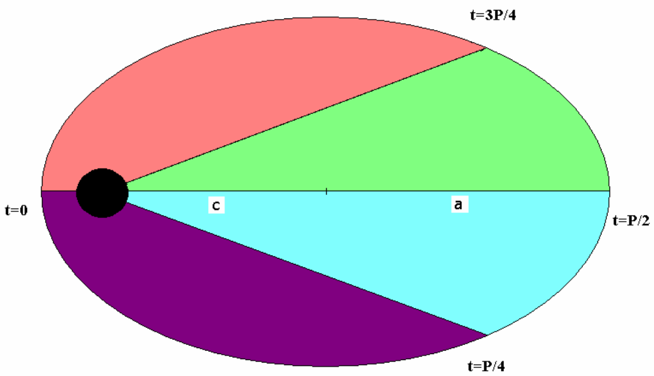
Kepler's laws : The motion of a star around its companion

1. Orbit = ellipse, with the companion star on a focus

2. Constant areal velocity

3. $a^3/P^2 = G/4\pi^2 (\mathcal{M}_1 + \mathcal{M}_2)$
   $\Leftrightarrow \quad {a_{UA}}^3/{P_{an}}^2 = (\mathcal{M}_1 + \mathcal{M}_2)/\mathcal{M}_\odot$

   The sum of the masses derived from the period, the apparent semi major axis and the parallax



The 2-body problem: Orbit Reconstruction

- Mean anomaly :
  $M = 2\pi\,(t - T_0)\,/\,P$
- Eccentric anomaly :
  $M = E - e \sin E$

$$E0 = M + \frac{e \sin M}{(1 - 2\,e \cos M + e2)^{1/2}}$$

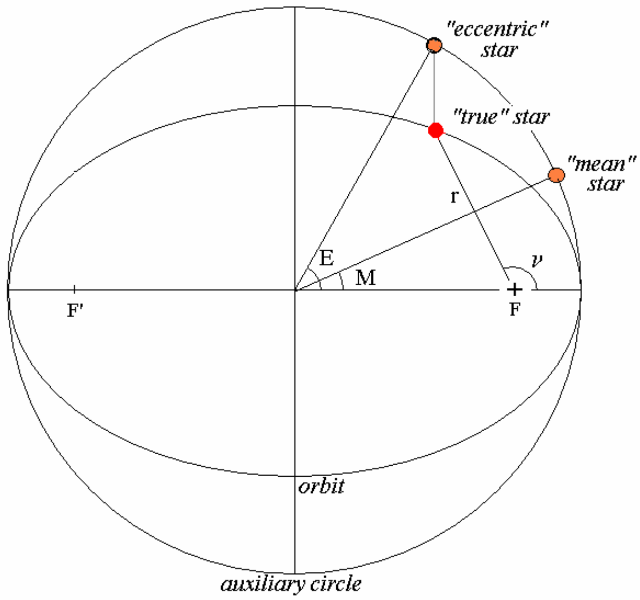
$M_0 = E_0 - e \sin E_0$

$E_1 = E_0 + (M - M_0)/(1 - e \cos E_0)$

$M_1 = E_1 - e \sin E_1$

etc …

- True anomaly :
  $1 + e \cos \nu = (1 - e^2) / (1 - e \cos E)$



$r = a\ (1 - e^2) / (1 + e \cos \nu)$

See exercise 2 for more info !

The 2-body problem: velocity

Relative velocity : v = $[G (\mathcal{M}_1 + \mathcal{M}_2) (2/r - 1/a)]^{1/2}$

- for a circular orbit : $v_{circ} = [G (\mathcal{M}_1 + \mathcal{M}_2) / a]^{1/2}$
- for parabola ($a = \infty$) : $v_{para} = [2 G (\mathcal{M}_1 + \mathcal{M}_2) / r]^{1/2}$, the maximum for a system gravitationally bounded

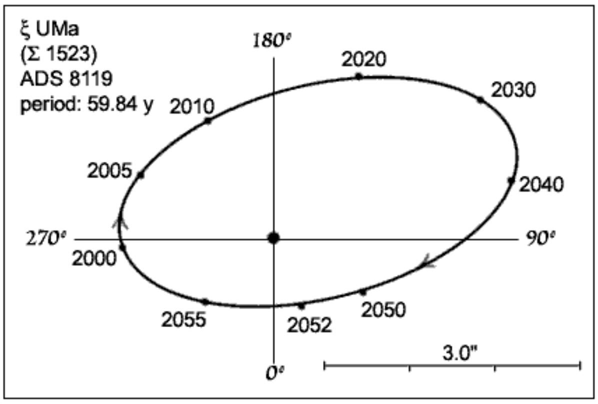
$\Rightarrow$ *Selection of physical binaries when the proper motions and the radial velocities of the components are known.*

Orbital Visual Binaries: history

1776 : Assuming double stars are due to perspective, William Herschell tried to use them for parallax measurements..

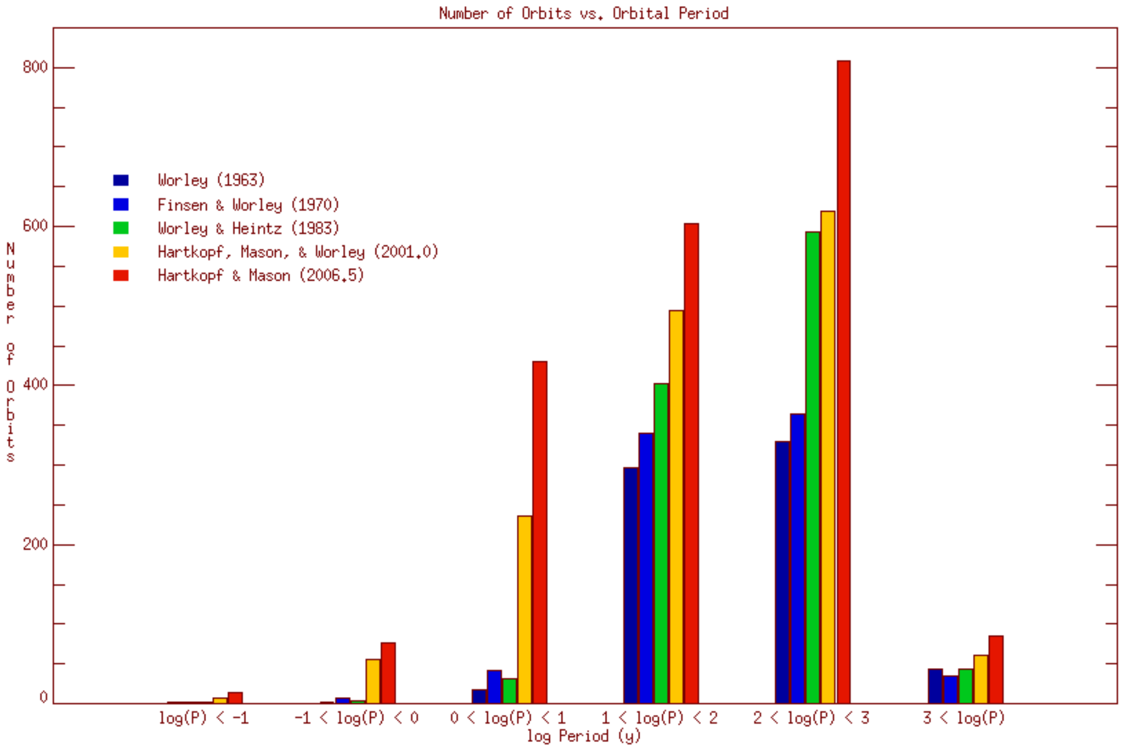
1803 : .. And he discovered that some of them (Castor, γ Vir) were orbital binaries

1827 : derivation of the orbit of ξ UMa by Félix Savary



2007 : 1888 binaries with orbits in the " 6th catalogue of visual binary orbits" (including a large proportion that were not observed on a complete period).

Number of orbits vs period



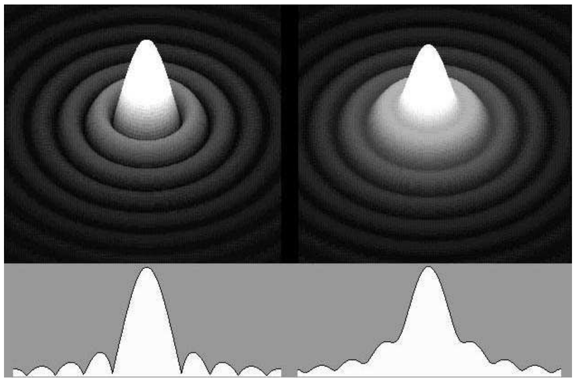
Angular resolution

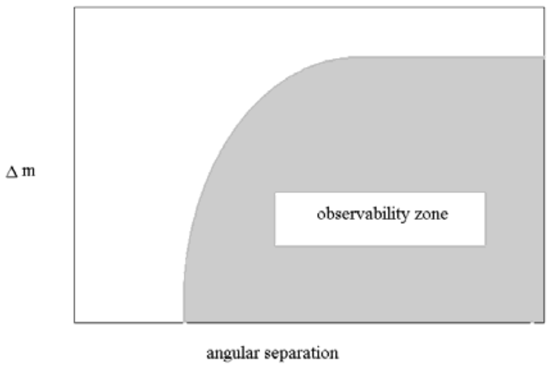
The apparent separation must be larger than the radius of the Airy disk :

$1.22\ \lambda/D$

In practice, for $\lambda = 0.5\ \mu$:

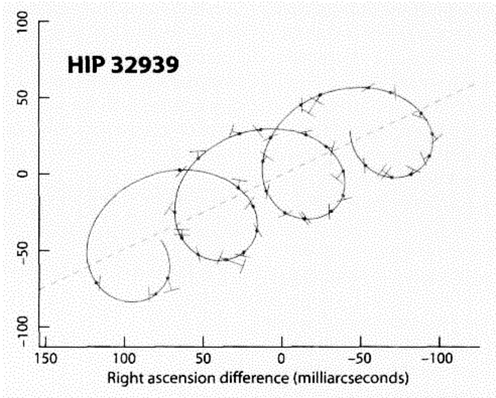
$\rho = 0.1''/D(m)$

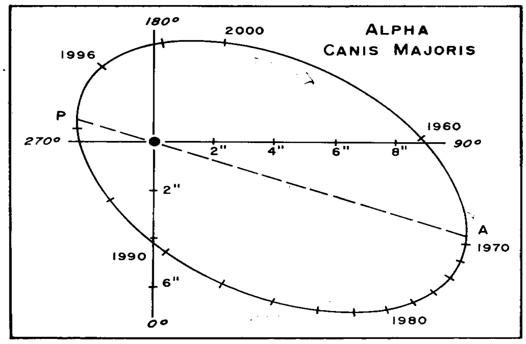




Moreover, the limit increases with the magnitude difference

Astrometric binaries





1844 : The first astrometric binaries are discovered by Bessel. Sirius and Procyon orbiting the barycentres of their systems (faint white dwarf companions).

Astrometric binaries

Semi major axes :

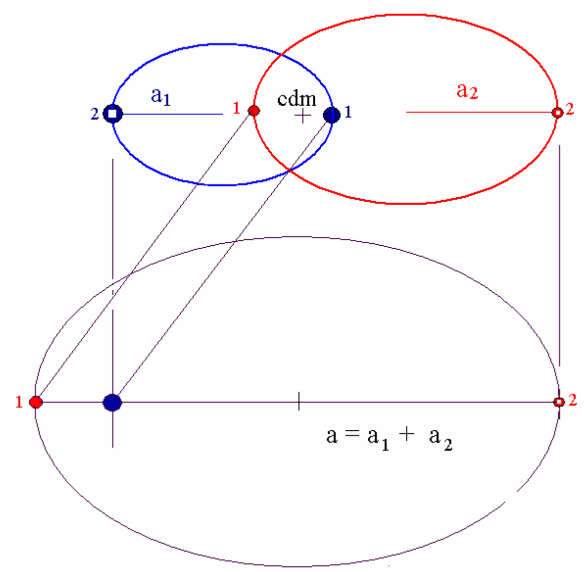
$a = a_1 + a_2$

Since $a_1/a_2 = \mathcal{M}_2/\mathcal{M}_1 = q$,

$a_1 = a.q/(1 + q)$

Kepler's 3rd law becomes :

$$\frac{a_1^3(\text{“})}{P^2(\text{an})} = \mathcal{M}_1 \frac{q^3}{(1+q)^2} D^{-3}(\text{pc})$$



Generally, a ***photocentre*** *is orbiting the barycentre*

The efficiency of astrometry

$a_{1('')} = [\mathcal{M}_{1(\mathcal{M}_\odot)} . P_{year}^{2}]^{1/3} \times [q/(1+q)^{2/3}] / D_{pc}$

With $\mathcal{M}_1 = 1\mathcal{M}_\odot$ *and* D *= 100 pc,* a *equals :*

P =	1 month	1 year	5 years	10 years
0.08 $\mathcal{M}_\odot$	145 µas	760 µas	2200 µas	3500 µas
0.01 $\mathcal{M}_\odot$	18 µas	95 µas	280 µas	440 µas

With Gaia, the accuracy of individual measurements will be around 70-100 µas for bright stars.

Spectroscopic binaries

1889: Miss Maury found that the lines in Mizar spectrum are sometimes double ⇒ spectroscopic binary, P=20j, $K_{1,2}$=68 km/s.

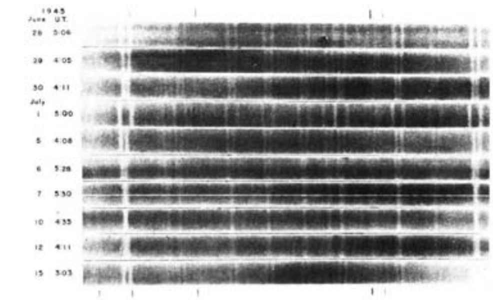
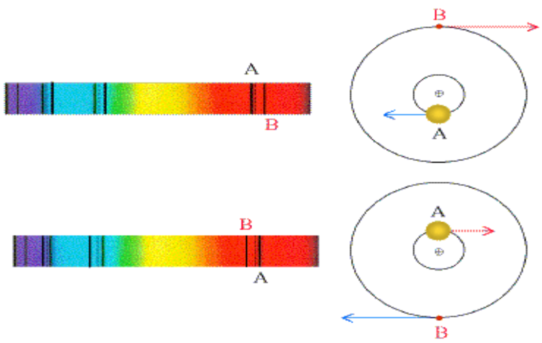


Fig. 9.5. From top to bottom a time sequence of the spectra of the binary star system Mizar is shown. Both stars are too close together to separate their spectra. The line systems of both stars are seen on each spectrum. The line positions of both stars shift sinusoidally due to the Doppler shift which results from the orbital motions. The wavelengths of the spectral lines of stars 1 and 2 vary in antiphase. (From Binnendijk 1960.)



The shift of the lines ⇒ the radial velocity : $V_R = c \,.\, \Delta\lambda/\lambda$

21 nov 2007 : 2653 *SBs with orbits in the SB9 catalogue*

Radial velocity of a SB (1)

We define

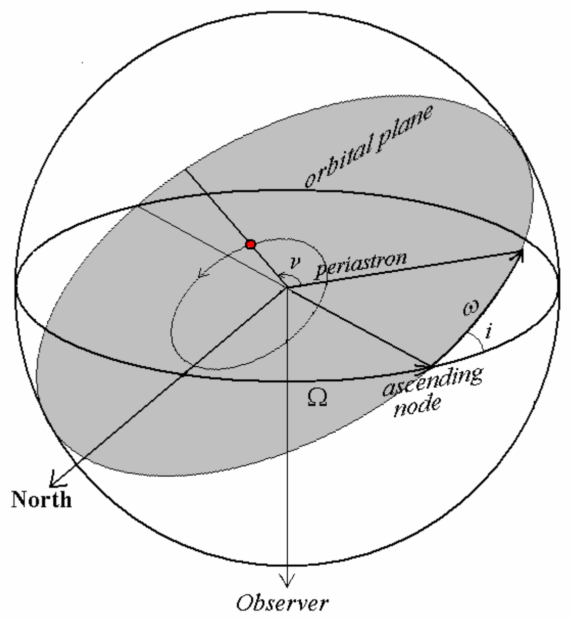
- *i : the inclination*
- *ω : the argument of periastron.*

Then, the RV is derived from Kepler's laws 1 and 2 :

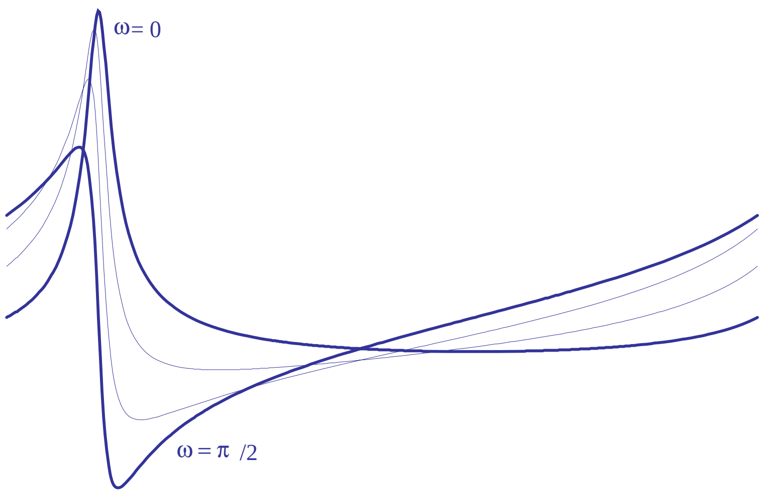
$V_R = V_\gamma + K_{1,2}[e \cos \omega + \cos(\omega+\nu)]$

With

$K_{1,2} = 2 \pi a_{1,2} \sin i / [P (1-e^2)^{1/2}]$



Radial velocity of a SB (2)



RV curves for e=0.8, assuming ω=0, 30, 60 and 90°

e and ω are derived from the shape of the velocity curve

Spectroscopic binaries : masses

Defining the mass ratio : $q = \mathrm{K}_1 / \mathrm{K}_2$ *, the 3rd Kepler's law gives :*

$$\mathrm{K}_1 = (2\pi.\mathrm{G}.\mathcal{M}_1/P)^{1/3} \times [q/(1+q)^{2/3}] \times \sin i \,/(1 - e\,)^{1/2}$$

$\Rightarrow$ *The "mass function" :*

$$f_{\mathcal{M}} = \mathrm{K}_1{}^3{}_{\mathrm{km/s}}(1 - e^2)^{3/2} P_{jours} \times 1{,}036\ 10^{-7}\ \mathcal{M}_\odot$$
$$= \mathcal{M}_1\, q^3 \sin^3 i \,/\, (1 + q)^2$$

$\Rightarrow$ $\mathcal{M}_{1,2} \sin^3 i$ *when* q *is known (SB2)*

Efficiency of spectroscopy

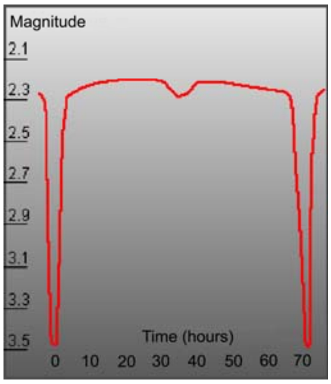
$$\mathrm{K}_1 = 212.9 \times (\mathcal{M}_{1/\odot}/P_{days})^{1/3} \times [q/(1+q)^{2/3}] \times \sin i \,/(1 - e\,)^{1/2}\ \mathrm{km/s}$$

With $\mathcal{M}_1 = 1\mathcal{M}_\odot$*,* $e = 0$ *and* $i = \pi/2$*,* K *is :*

$\mathcal{M}_2$ P	3 days	1 month	1 year	5 years
0.7 $\mathcal{M}_\odot$	73 km/s	34 km/s	15 km/s	9 km/s
0.08 $\mathcal{M}_\odot$	11.2 km/s	5.2 km/s	2.3 km/s	1.3 km/s
0.01 $\mathcal{M}_\odot$	1.4 km/s	0.65 km/s	0.28 km/s	0.17 km/s

For Gaia, the errors of individual measurements will be $E_{RV} \geq 1$ *km/s, but 1 m/s may be achieved with on-ground observations*

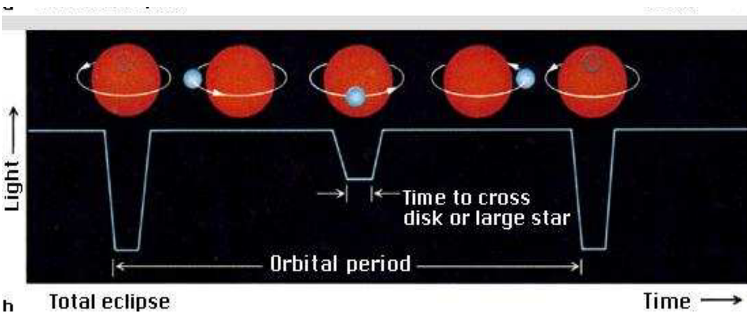
Eclipsing binaries

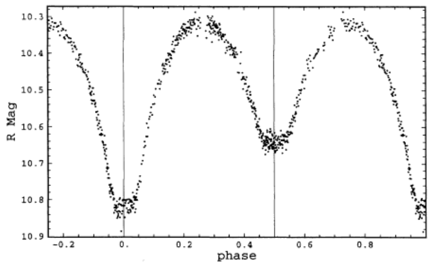


1669 : Montanari noticed Algol variabililty

1783 : Goodricke interpretation: a cloud orbiting Algol with a 69 h-period; luminosity decrease :1,3 mag in 4h ½.

Eclipsing binaries (2)



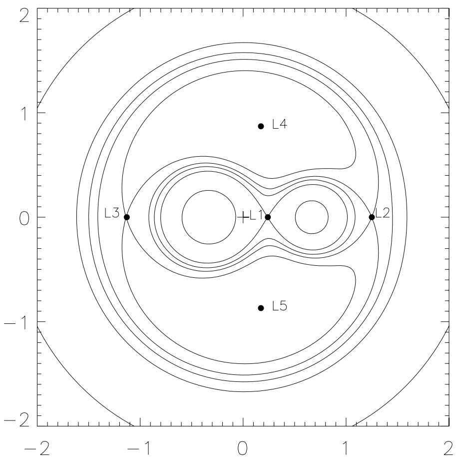


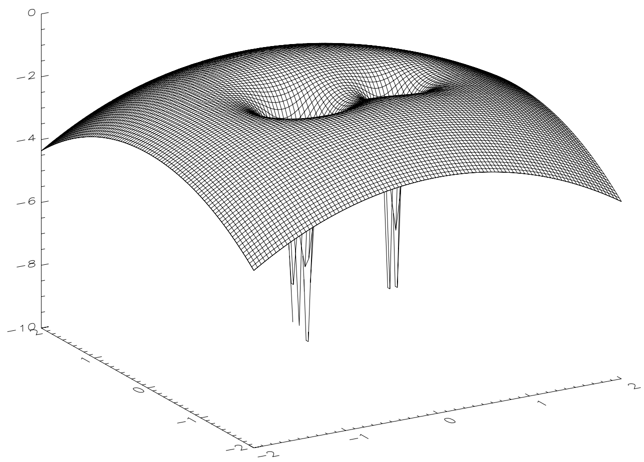
Rare: i close to $\pi/2$.

Light curve smoothed by :

- *limb darkening*
- *mutual irradiation*
- *tidal deformations of the stars*

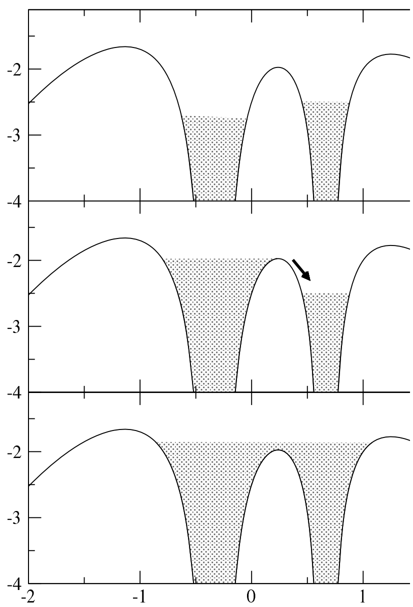
Roche's potential (1)

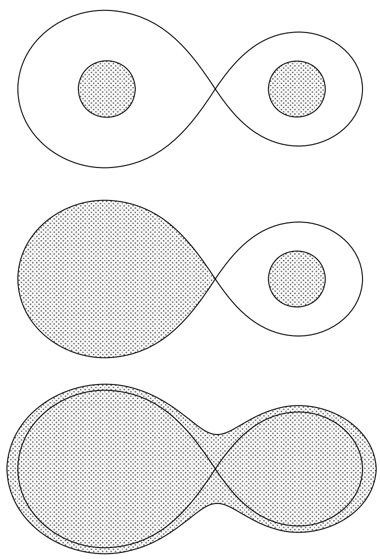




Potential in the rotating reference frame of a binary system

Roche





Detached system

Semi-detached

ex:. Algol

Contact system

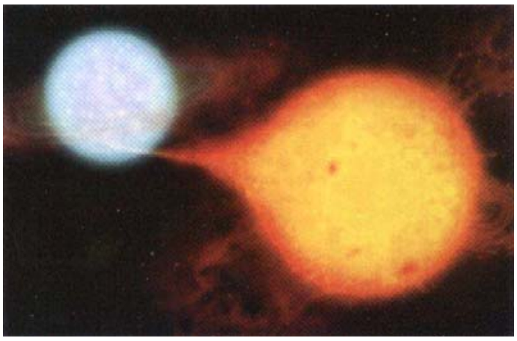
ex.: W UMa

"Radius" of the Roche's

Algol paradox

Primary : B8 V*;* $\mathcal{M}_1 = 3.7\ \mathcal{M}_\odot$,
R_1=2.02 Gm, T_1=13000 K

Secondary : K2 IV*;* $\mathcal{M}_2 = 0.8\ \mathcal{M}_\odot$,
R_2=2.45 Gm, T_2=4500 K, Δm=2.5



A young primary with an old under luminous secondary ??

The present secondary was the most massive component at the origin. When it became giant, a part of its mass was transferred to the companion

Summary

Info\Binaries	Visual orbit	Astrometric B.	Spectroscopic B.	Eclipsing B
P min *Max*	~ 1 y ~ 200 y	~ 1 y ~ 100 y	A few hours ~ 30 y	~1 h *A few days*
inclination ?	*yes*	*yes*	*no*	*yes*
a	*apparent*	a_1 *or photocentre*	$a_1 \sin i$ (km) $a_2 \sin i$	~ a/R
masses	$(\mathcal{M}_1+\mathcal{M}_2) \times D^{-3}$	$\mathcal{M}_1\ q^3/(1+q)^2$ (for large Δm)	$\mathcal{M}_1\ (q\sin i)^3/(1+q)^2$ $\mathcal{M}_{1,2}\ \sin^3 i$ (SB2)	
luminosity	Δm		~ Δm for SB2	$\delta m \Rightarrow \Delta m$
Number	1888		2653	~4000

Merging SB and AO: the brown dwarf candidate GJ 483

GJ483 : SB1, P=271 d, e=0,784, ω = 264°, K_1=1,07 km/s, K3 V $\Rightarrow$ $\mathcal{M}_1$=0,72 $\mathcal{M}_\odot$

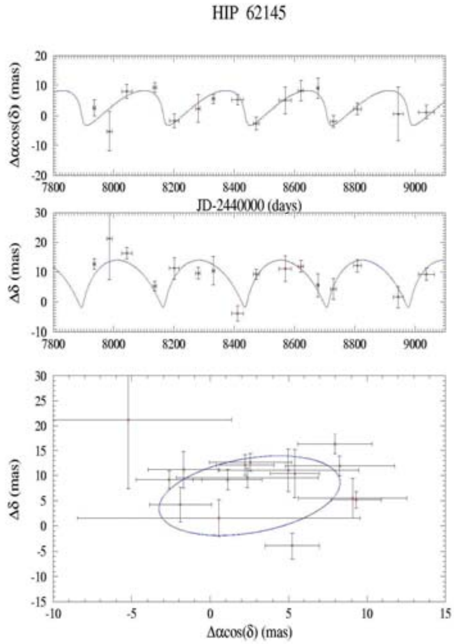
$\Downarrow$

$\mathcal{M}_2 \sin i = 0{,}016\ \mathcal{M}_\odot$
$a_1 \sin i$ = 2,475 Gm

+

Hipparcos:
a_1 = 8,39 mas (astrometric orbit)
ϖ = 67,47 mas $\Rightarrow$ $i = 8°$

$\Rightarrow \mathcal{M}_2 = 0{,}137\ \mathcal{M}_\odot$



Merging SB and AO

Combining Gaia astrometry and accurate on-ground RV for SB2

$\Downarrow$

Masses of components with errors ~ 1 %

Binary stars : statistical properties

Relevant parameters :

- *Mass ratio,* $q=\mathcal{M}_2/\mathcal{M}_1$*, ou* $\Delta m = m_2 - m_1$.
- *period, or semi-major axis*
- *eccentricity*

Distributions ? Correlations between parameters ?

Problems :

- *Derivation of the distributions (f(q) of SB1 ..)*
- *Selection effects evaluation*
- *Bias correction (simulation, direct methods ..)*

Selection effect : contribution of the secondary to total luminosity

For a sample of binaries with the same primary spectral type and a limiting magnitude:

$m_{1+2} = m_1 - 2,5 \log(1 + 10^{-\Delta m/2,5})$

For MS stars:

$\Delta m \sim - k \log q$

Since

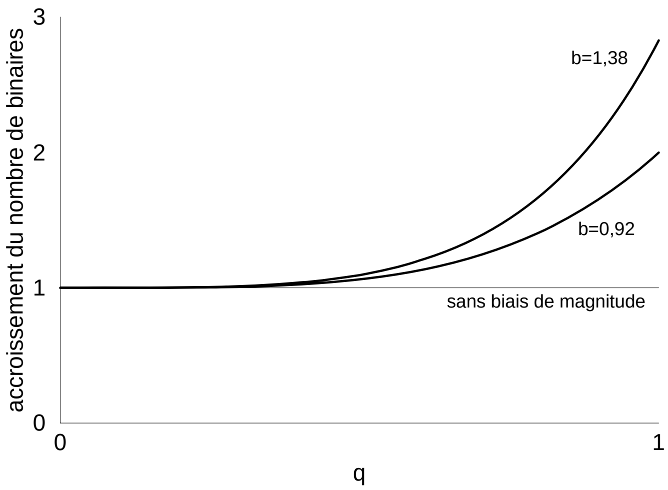
$f(m_1) \sim \exp(b.m_1)$

(b=0.92 *to* 1.38),

The proportion of binaries with mass ratio q *is increased with coefficient:*

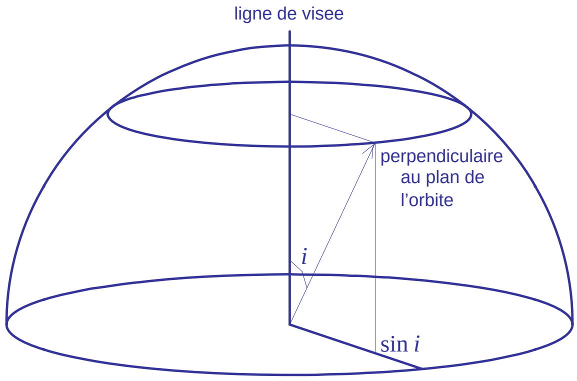
$(1 + q^{k/2,5})^{2,5\, b/\mathrm{Ln}\, 10}$

= 2 *to* 2,83 *for* q=1



Selection effects : inclinations of SB orbits

For a spectroscopic orbit, $i \in [0, \pi/2]$



The angular momentums are isotropically oriented $\Rightarrow$ probability to get $i \pm di/2$:

$f_i(i)\, di \propto 2\pi \ \sin i\, di$

therefore :

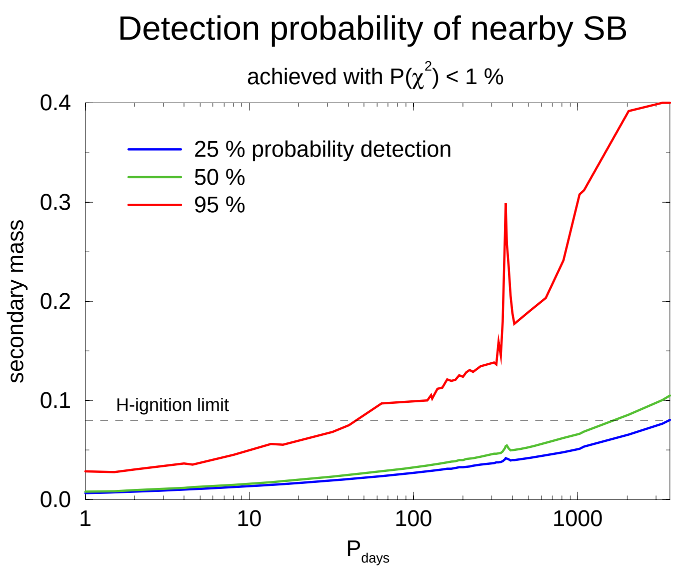
$f_i\,(i) = \sin i$

Selection effects for SB: secondary masses and periods

SB detection depends on :

- $K = f(q, P) > E_{RV}$
- P *, and observations sampling*

Detection based on σ_{RV}/E_{RV}, *with threshold* $P(\chi^2) < 1\ \%$.



Distribution of q for SB1

From observations: $Y = f_{\mathcal{M}} / \mathcal{M}_1 = q^3 \sin^3 i \,/\, (1+q)^2$

$\Rightarrow$ q *between* q(sin i =1) *et* q(SB2).

*Simple, but **wrong**:* $\sin^3 i = \langle \sin^3 i \rangle \Rightarrow$ q *for each SB1*

Iterative Richardson-Lucy algorithm :

Assuming an a priori f(q), *convert each* Y *in a probability distribution of* q, $\varphi(q|Y)$:

$\varphi(q|Y) \propto f(q) \sin i(q|Y) \tan i(q|Y)$, *avec* $\int \varphi(q|Y) = 1$

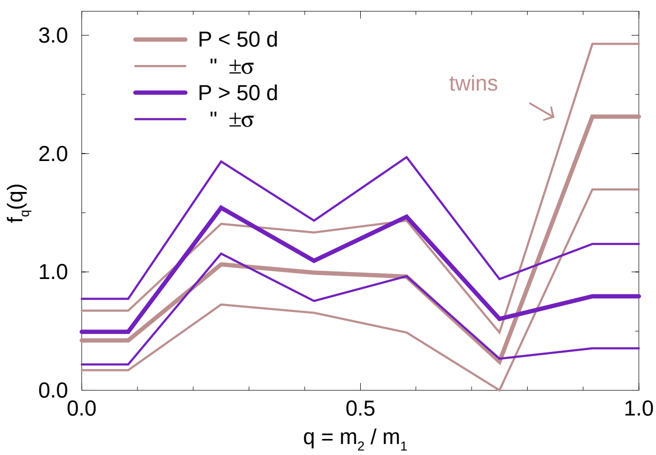
And then: $f(q) = \Sigma\, \varphi(q|Y) / N$

f(q) for binaries with P< 10 y

For G-K-M primaries, 2 modes:

• 0,1 < q < 0,7 ,

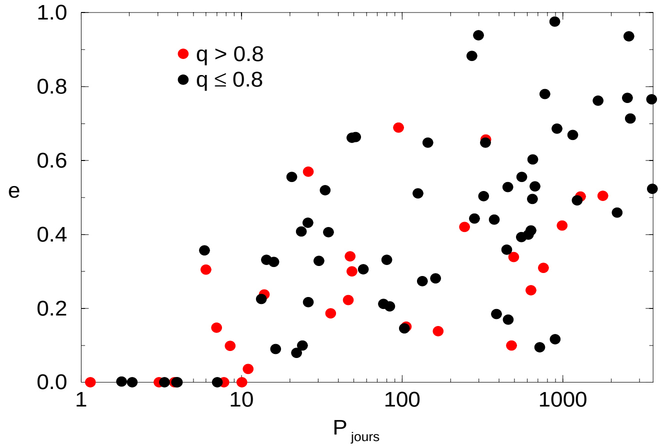
With a "brown dwarf desert" (q<0,1) vanishing for P> 2-3 years



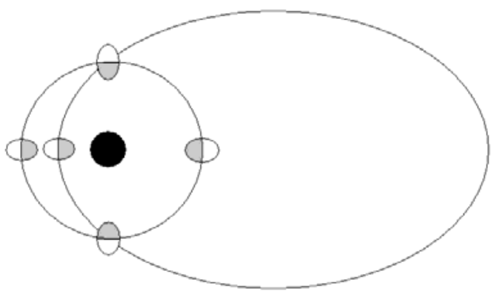
• q > 0.8, with a peak around q=1 ("twins"). Vanishing when P increasing.

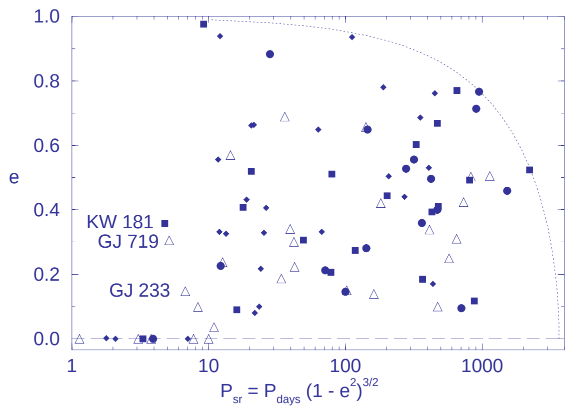
Eccentricity distribution

- *Short period orbits are all circular.*
- *Long period orbits may be very eccentric*
- *Orbits of twins are less eccentric (for G-K MS primary).*



Tidal circularisation





Circularisation : Loss of Energy, but the angular momentum of the orbit is almost constant (except for very close binaries)

$\Rightarrow$*Constant semi latus rectum (* $h = a(1-e^2)$ *) of the orbit*

Tidal torque $\propto 1/r^6 \Rightarrow$ *Dramatic transition between circularized orbits and unchanged orbits.*

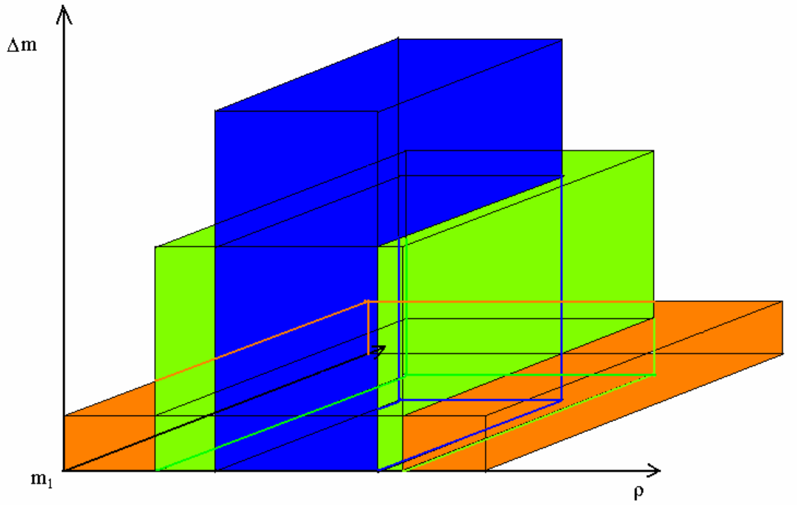
Visual binaries: f(q) *(1)*

Bias (ρ,Δm,m_1)

f(Δm) *or* f(q) *derived with the method of the "nested boxes" :*

(ρ, m_1) ⇒ q limite

⇓

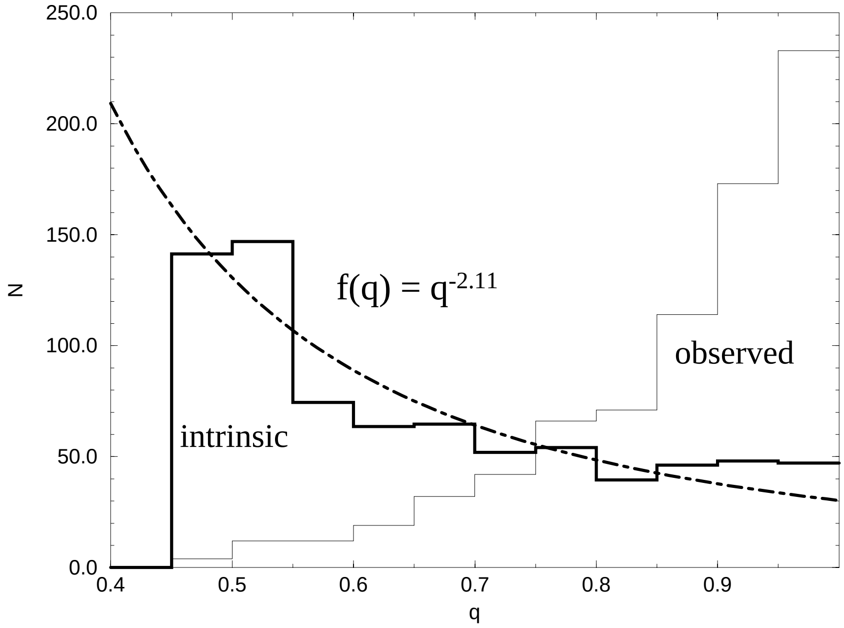


Several f(q > q_{lim}) *which are merged thanks to their overlapping parts.*

Visual binaries: f(q) *(2)*

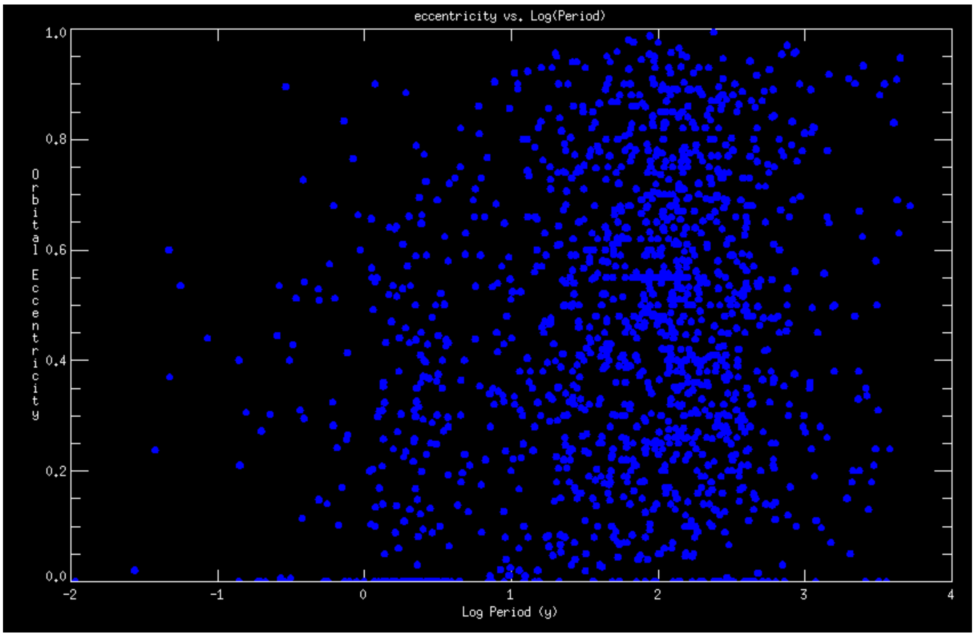
For short-period VB, f(q) *is bimodal like for SB*

When P ≥1000 *years,* f(q) *similar to the initial mass function (IMF).*



The distribution of q derived from CPM stars

Eccentricity distribution



$f(e) = (e - e^2)^{1/2}$

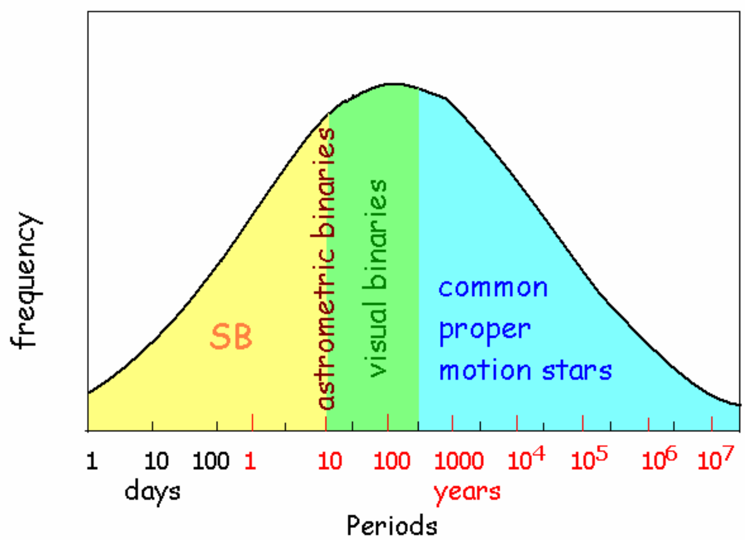
Frequency and period distribution

F7-K dwarfs: 56 % binaries. M dwarfs: 26 % binaries

Log-Normal distribution from 1 day to 10 millions years

$\langle \log P_{days} \rangle = 4.8$

$\sigma_{\log P} = 2.3$



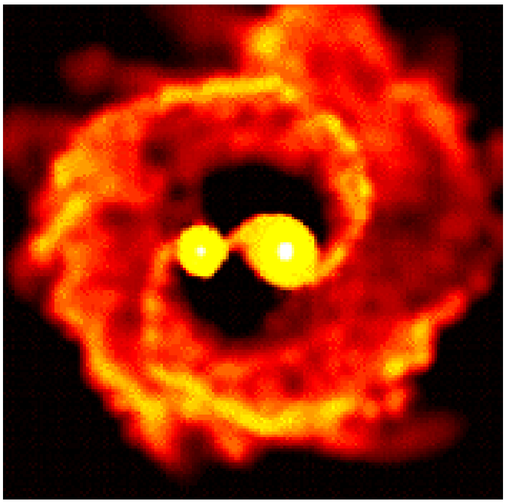
M dwarfs: less systems with a > 10 AU

Formation of binary stars

Two kinds of processes :

***Fragmentation** of a collapsing cloud, or of a disk.*

***Capture** of two forming stars, energy excess evacuated by the accretion disk.*



***The main difficulty:** formation of close systems*

Collapse of an ISM cloud

Jeans condition:

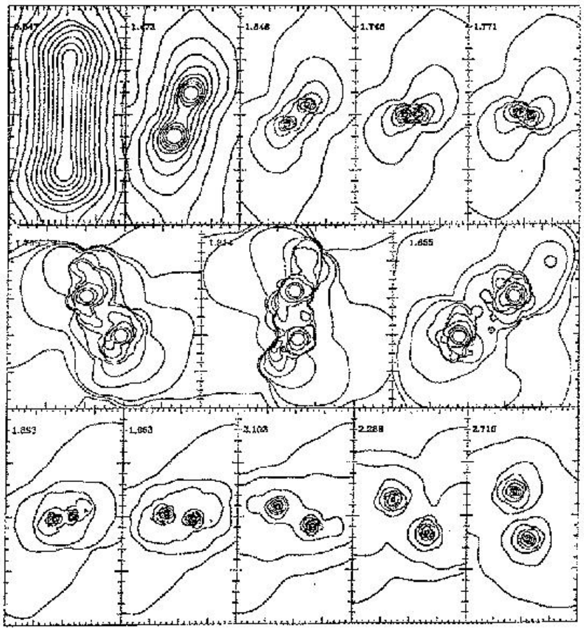
$|E_{\mathrm{grav}}| > E_{\mathrm{therm}}$
$(+E_{\mathrm{rot}}+E_{\mathrm{turb}}+E_{\mathrm{magn}} \sim 0)$

$\Downarrow$

$\mathcal{M} > \mathcal{M}_{\mathrm{Jeans}} \Leftrightarrow R > R_{\mathrm{Jeans}}$

$$R_{Jeans} = \sqrt{\frac{5\mathrm{R}T}{2\mathrm{G}\mu} \times \frac{3}{4\pi\rho}}$$

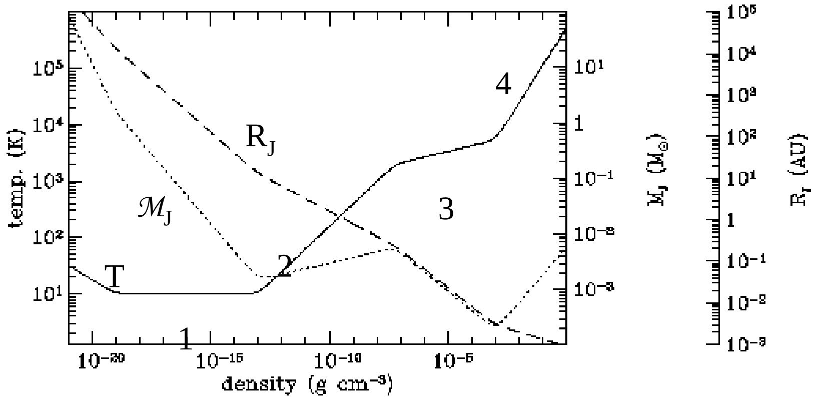
R: ideal gas constant
***T*: temperature**
G: gravitation constant
μ: molecular weight
ρ: density



Fragmentation of a collapsing cloud

4 phases :

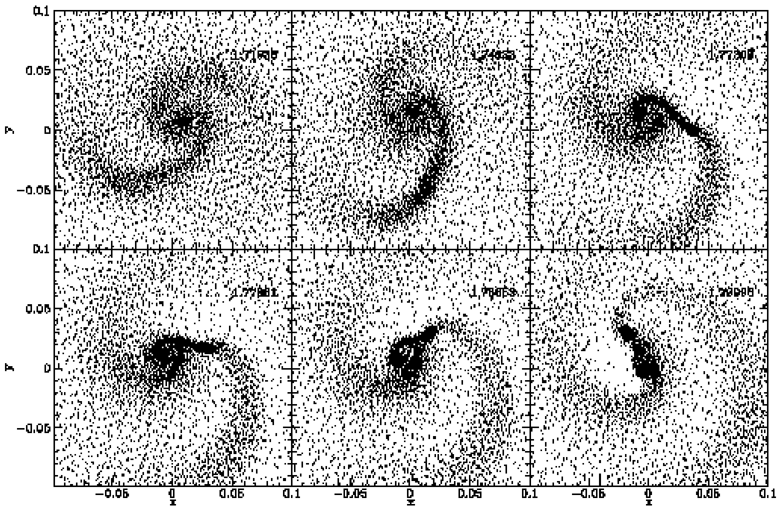
1. isotherm ⇒ a 1 UA core.
2. quasi-static (due to opacity)
3. $H_2 \rightarrow 2H$, nearly isotherm
4. quasi-static



Fragmentation is possible at the end of phases 1 and 3; after phase 3, $\mathcal{M}_{Jeans} < 10^{-3} \mathcal{M}_\odot$ ⇒ final q =1 and a = 0.01 UA

Pb : to get a unstable core after phase 3, since it should still have been broken at the end of phase 1 ....

Disk fragmentation



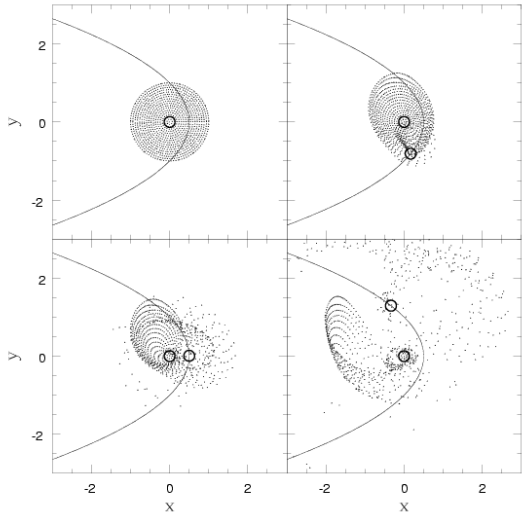
Rotational instability fragmentation

Capture with dispersion of circumstellar disk

Ejection of the outer part of the disk

⇓

capture



Leads to : small q, long P and large e

Conclusion : the present

The statistical properties are reasonably known for solar-type stars, less for M-type dwarfs, and poorly known for early-type dwarfs .

The distribution of mass ratios suggests several formation processes

The distribution of periods suggests a unique formation process

The models hardly generate binaries with P < 1 year.

Conclusion: the future

Gaia will highly improve our knowledge of the statistical properties of binaries

.. and the models will certainly follow the progresses of the statistics !!